Minisymposium 2

Numerics for PDE-Constrained Control Problems

Leiterin des Symposiums:

Prof. Dr. Angela Kunoth
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Universität Bonn
Wegelerstraße 6
53115 Bonn, Germany
Donnerstag, 21. September
Raum 610, Institut für Angewandte Mathematik, Wegelerstr. 6

15:00 – 15:50 Helmut Harbrecht *(Kiel, z.Zt. Bonn)*
Shape Optimization for Elliptic PDEs

16:00 – 16:50 Max D. Gunzburger *(Florida State University)*
Improved construction and application of reduced-order models

17:00 – 17:50 Karl Kunisch *(Universität Graz)*
Reduced Order Control Based on Approximate Inertial Manifolds

Freitag, 22. September
Raum 610, Institut für Angewandte Mathematik, Wegelerstr. 6

15:00 – 15:50 Ekkehard Sachs *(Trier & Virginia Tech)*
t.b.a.

16:00 – 16:20 Roland Pabel *(Bonn)*
Wavelet Methods for PDE Constrained Elliptic Control Problems with Dirichlet Boundary Control

16:30 – 16:50 Thomas Slawig *(TU Berlin)*
PDE-Constrained Control Using Commercial Simulation Software – Control of the Navier-Stokes Equations with FEMLAB

17:00 – 17:50 Steve Hou *(Iowa State University)*
t.b.a.
Shape optimization is quite indispensable for designing and constructing industrial components. Many problems that arise in application, particularly in structural mechanics and in the optimal control of distributed parameter systems, can be formulated as the minimization of functionals defined over a class of admissible domains. The present talk aims at surveying on shape optimization for elliptic boundary value problems. Especially, the following items will be addressed:

- analysis of shape optimization problems,
- the discretization of shapes,
- first and second order shape optimization methods,
- existence and convergence of approximate shapes,
- efficient numerical techniques to compute the state equation.

The approximate solution of optimization and control problems constrained by nonlinear partial differential equations is often a formidable task. At the least, it requires multiple state solutions or, in the feedback control setting, real-time state solutions. These tasks are difficult or even impossible to accomplish without using some sort of model reduction technique. We discuss some recent developments in the construction and application of reduced-order models for reducing the cost of state solutions. We focus on two issues: the generation of snapshots upon which reduced-order models are built and the incorporation of a parallelism-in-time methodology into reduced-order solution strategies. In the first case, we discuss some novel techniques developed by several authors (notably Antony Patera and co-workers and Omar Ghattas, Karen Willcox, and co-workers) for adaptively sampling parameter space during the snapshot generation process. We also show the results of some computational experiments that test the ability of reduced-order models to remain useful as the dynamical nature of the state solution changes during the optimization process. We close by describing some recent
work (by Janet Peterson and co-workers) on combining the parareal algorithm for time parallelism with reduced-order modeling, showing that the combination results in significant speedups compared to using only reduced-order modeling.

**Karl Kunisch  (Universität Graz)**

**Reduced Order Control Based on Approximate Inertial Manifolds**

A reduced-order method for optimal control problems in infinite-dimensional based on approximate inertial manifolds is developed. Convergence of the cost, optimal controls and optimal states of the finite dimensional, reduced-order, optimal control problems to the original optimal control problem is analyzed. Special attention is given to the particular case when the dynamics are described by the Navier Stokes equations in dimension two. – This is joint work with Prof. Ito, North Carolina State University.

**Ekkehard Sachs  (Trier & Virginia Tech)**

*t.b.a.*

*(Abstrakt lag bei Redaktionsschluss noch nicht vor.)*

**Roland Pabel  (Bonn)**

**Wavelet Methods for PDE Constrained Elliptic Control Problems with Dirichlet Boundary Control**

We consider wavelet methods applied to control problems constrained by a linear elliptic PDE with Dirichlet boundary control. In order to handle the latter in a convenient way, we employ a saddle point formulation for the PDE constraints. Then the necessary conditions for optimization lead to a coupled system of saddle point problems. We investigate fast iterative solution methods for this system with optimal preconditioners based on the Fast Wavelet Transform for problems on up to three-dimensional spatial domains. In particular, the choice of different modelling parameters in the cost functional and their effect on the numerical simulation and solution will be discussed.
We show how the commercial simulation software FEMLAB can be used to solve PDE-constrained optimal control problems. We give a general formulation for such kind of problems and derive the adjoint equation and optimality system. Then these preliminaries are specified for the stationary Navier-Stokes equations with distributed and boundary control. The main steps to define and solve a PDE with FEMLAB are described. We describe how the adjoint system can be implemented, and how the optimality system can be used by FEMLAB’s built-in functions. Special crucial topics concerning efficiency are discussed. Examples with distributed and boundary control for different type of cost functionals in 2 and 3 space dimensions are presented.