S4D2 – Graduate Seminar on Topology

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1 Fibrations

Definition of fibration (Hurewicz fibration, Serre fibration), examples, the long exact sequence in homotopy groups, pathspaces and loopspaces.
References: [Hat02, p. 375 – 380 and p. 405-409] and [tD08, Sections 5.5 and 6.3]

2 Eilenberg–MacLane spaces

Definition, Ω-spectrum, representability of cohomology by Eilenberg–MacLane spaces, cup-products.
References: [Hat02, p. 365 – 366 and p. 393 – 405]

3 Postnikov Towers

Definition and existence of Postnikov towers, the k-invariant, Moore-Postnikov towers.
References: [Hat02, p. 410 – 415]

4 Obstruction theory

Obstructions for the existence of lifts along fibrations.
References: [Hat02, p. 415 – 419] and [Bre97, Section VII.13]

5 Classifying spaces

Classifying spaces for topological groups and examples.
References: [tD08, Sections 14.1,14.3 and 14.4]

6 Vector bundles

Reminder on the definition, pullbacks of vector bundles, classification of vector bundles.
References: [tD08, Sections 14.2,14.4 and 14.5]

7 Steenrod squares

References: [Bre97, Sections VI.15 and VI.16]

8 Characteristic classes

References: [Bre97, Sections VI.17]

9 Brown representability

Every cohomology theory can be represented by a spectrum.
References: [Hat02, Section 4.E]

10 Spectra and homology theories

Stable homotopy groups as a reduced homology theory, homology theories from spectra.
References: [Hat02, Section 4.F]

11 Spanier–Whitehead duality

References: [Swi02, Chapter 14]

12 Bordism as a homology theory

Basic properties of bordism groups, bordism forms a homology theory.
References: [tD08, Section 21.1] and [BtD70, II. Kapitel]

13 Pontrjagin–Thom and Thom spectra

Relation between bordism groups and homotopy groups of Thom spaces (Pontrjagin–Thom), definition of Thom spectra.
References: [tD08, Section 21.2 and 21.3], [BtD70, III. Kapitel] and [Bre97, Section II.16]

14 Homology with local coefficients

References: [Hat02, Section 3.H]

References

[Bre97] Glen E. Bredon, *Topology and geometry.*, corr. 3rd printing of the 1993 original ed., Grad. Texts Math., vol. 139, Berlin: Springer, 1997.

[BtD70] T. Bröcker and Tammo tom Dieck, *Kobordismentheorie*, Lect. Notes Math., vol. 178, Springer, Cham, 1970.

[Hat02] Allen Hatcher, *Algebraic topology*, Cambridge: Cambridge University Press, 2002.

[Swi02] Robert M. Switzer, *Algebraic topology – homology and homotopy.*, reprint of the 1975 edition ed., Class. Math., Berlin: Springer, 2002.

[tD08] Tammo tom Dieck, *Algebraic topology*, EMS Textb. Math., Zürich: European Mathematical Society (EMS), 2008.