

Exercises for Algebraic Topology I – Sheet 3

Uni Bonn, WS 2018/19

Aufgabe 9. Let X be a G -CW-complex. Show that X is free if and only if $C_*(X)$ is a free $\mathbb{Z}G$ -chain complex.

Aufgabe 10. Prove or disprove that a principal G -bundle $p: E \rightarrow B$ is trivial if and only if there is a map $s: B \rightarrow E$ with $p \circ s = \text{id}_B$.

Aufgabe 11. Let $p: ES^1 \rightarrow BS^1$ be the universal principal S^1 -bundle. Prove or disprove that ES^1 is not S^1 -homotopy equivalent to a finite-dimensional S^1 -CW-complex.

Aufgabe 12. Let G_0 and G_1 be topological groups. Let $\text{pr}_i: G_0 \times G_1 \rightarrow G_i$ be the projection for $i = 0, 1$. Prove or disprove that the map

$$B\text{pr}_0 \times B\text{pr}_1: B(G_0 \times G_1) \rightarrow BG_0 \times BG_1$$

is a homotopy equivalence.