Exercises for Algebraic Topology I – Sheet 12

Uni Bonn, WS 2018/19

Aufgabe 45. Consider two closed oriented connected manifolds M and N. Let $f\colon M \to N$ be a map of degree one. Suppose that f is covered by a map of bundles which is fiberwise an isomorphism

$$\begin{tikzcd} TM \arrow[r, "\overline{f}"] \arrow[d] & TN \arrow[d] \\ M \arrow[r, "f"] & N \end{tikzcd}$$

Prove or disprove that $\chi(M) = \chi(N)$ holds.

Aufgabe 46. Compute the first Chern class or the first Stiefel-Whitney class of the universal 1-dimensional complex or real vector bundle respectively.

Aufgabe 47. Let ξ be a real vector bundle with projection $p\colon E \to B$. Let $p_0\colon E_0 \to B$ be the restriction of p to E_0 which is obtained from E by removing the zero-section. Compute the Euler class of the pull back $p_0^*\xi$.

Aufgabe 48. Let G be an abelian group and $n \geq 1$. Let $p\colon X \to K(G, n+1)$ be a fibration such that X is contractible. Let $q\colon E \to B$ be a fibration over a CW-complex B with typical fiber $K(G,n)$. Which of the following assertions are true?

(a) The fibration p exists and its typical fiber is weakly homotopy equivalent to $K(G,n)$;

(b) The primary obstruction $\gamma(q)$ for finding a section for q exists and takes values in $H^{n+1}(B;G)$;

(c) Consider a map $f\colon B \to K(G, n+1)$. Show that the canonical isomorphism

$$[B, K(G, n+1)] \xrightarrow{\cong} H^{n+1}(B;G), \quad [f] \mapsto f^*\iota_{K(G,n+1)}$$

sends f to the primary obstruction of the pullback f^*p;

(d) Prove or disprove that two fibrations over B with typical fiber $K(G,n)$ which are obtained from p by pulling back with some map from B to $K(G, n+1)$ are strongly fiber homotopy equivalent if and only if their primary obstructions in $H^{n+1}(B;G)$ agree.

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handover on Wednesday, 15.01 in the lecture