

# Exercises for Topology I – Sheet 8

University of Bonn, WS 2018/19

**Exercise 29.** Construct a  $CW$ -structure on  $S^2$  with one 0-cell, one 1-cell and two 2-cells. Compute its cellular chain complex.

**Exercise 30.** Let  $X$  be a finite simplicial complex. Show that it inherits the structure of a finite  $CW$ -complex such that there is a bijective correspondence between the  $p$ -simplices of  $X$  and the  $p$ -cells of its  $CW$ -structure and that the simplicial and the cellular  $\mathbb{Z}$ -chain complexes are isomorphic.

**Exercise 31.** Let  $X$  be a compact  $CW$ -complex. Show that its singular homology with  $\mathbb{Z}$ -coefficients is finitely generated in each dimension. Construct a compact space for which this is not true.

**Exercise 32.** Let  $X$  be a finite  $CW$ -complex. Denote by  $\beta_n$  be the number of its  $n$ -cells. Let  $\mathcal{H}_*$  be a homology theory with values in  $\mathbb{Q}$ -modules which satisfies the (modified) dimension axiom and  $\dim_{\mathbb{Q}}(\mathcal{H}_0(\{\bullet\})) = d$ . Show

$$\begin{aligned}\sum_{n \geq 0} \dim_{\mathbb{Q}}(\mathcal{H}_n(X)) &\leq d \cdot \sum_{n \geq 0} \beta_n; \\ \sum_{n \geq 0} (-1)^n \dim_{\mathbb{Q}}(\mathcal{H}_n(X)) &= d \cdot \sum_{n \geq 0} (-1)^n \cdot \beta_n.\end{aligned}$$

**Please note the following announcement from the Fachschaft Mathematik:**

On December 3rd 7 pm (st) in the Lipschitz hall the Fachschaft (student council) organizes a Ladies Night for all female students enrolled in the Bachelor's program for mathematics (5th semester or higher) or in the Master's program. In case of questions, you can contact us via [gleichstellung@fsmath.uni-bonn.de](mailto:gleichstellung@fsmath.uni-bonn.de)

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to be handed in on 03.12. during the lecture