Exercises for Topology I – Sheet 2

University of Bonn, WS 2018/19

Exercise 5. Let $\mathcal{H}_*$ be a homology theory. Prove that $\mathcal{H}_n(S^1 \vee S^1 \vee S^2) \cong \mathcal{H}_n(T^2)$ for all $n \in \mathbb{Z}$. Prove that $S^1 \vee S^1 \vee S^2$ and T^2 have non-isomorphic fundamental groups and hence are not homotopy equivalent.

Exercise 6. Let $p(z)$ be a complex valued polynomial which has no roots on $S^1 = \{z \in \mathbb{C} \mid ||z|| = 1\}$ and m roots (counted with multiplicity) on $\{z \in \mathbb{C} \mid ||z|| < 1\}$. Let $f\colon S^1 \to S^1$ be given by $z \mapsto \frac{p(z)}{||p(z)||}$. Let $\mathcal{H}_*$ be a homology theory satisfying the dimension axiom.

Prove that $\mathcal{H}_1(f)\colon \mathcal{H}_1(S^1) \to \mathcal{H}_1(S^1)$ is given by multiplication with m.

Exercise 7. Let $f\colon S^n \to S^n$ be a fix-point free map for some $n \in \mathbb{Z}$, $n \geq 1$. Let $\mathcal{H}_*$ be a homology theory satisfying the dimension axiom. Prove that

$$\mathcal{H}_n(f) = (-1)^{n+1} \cdot \mathrm{id}\,.$$

Exercise 8. Let x and y be points in $\mathbb{R}^n_+ = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid x_1 \geq 0\}$ with neighborhoods U and V, respectively. Assume there is a homeomorphism $(U, x) \xrightarrow{\cong} (V, y)$. Prove that either both x and y are in the boundary $\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid x_1 = 0\}$ or both x and y are in the interior $\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid x_1 > 0\}$.

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to be handed in on 22.10. during the lecture