

Exercises for Topology I – Sheet 11

University of Bonn, WS 2018/19

Exercise 41. Show that $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$ have isomorphic cellular homology and cellular cohomology but their cellular cohomology rings are different.

Exercise 42. Let $f: \mathbb{C}P^\infty \rightarrow \mathbb{C}P^\infty$ be any map. Prove or disprove for cellular cohomology and every commutative ring R that $H^n(f; R): H^n(\mathbb{C}P^\infty; R) \rightarrow H^n(\mathbb{C}P^\infty; R)$ is bijective for all $n \geq 0$ if and only if it is bijective for $n = 2$.

Exercise 43. For which $d \geq 1$ is $\mathbb{C}P^d$ homotopy equivalent to the suspension of some topological space?

Exercise 44. Is the first cellular cohomology $H^1(X, A; \mathbb{Z})$ a torsionfree \mathbb{Z} -module for every CW -pair (X, A) ? Answer the same question for the first cellular homology $H_1(X, A; \mathbb{Z})$.