

# Topology I: Basic algebraic and categorical notions

FLORIAN KRANHOLD

- (I) **Groups, rings and fields** [Lan02, I–II]
  - groups, normal subgroups, factor groups, direct products,
  - free product, free groups, group presentations, abelianisation,
  - rings, integral domains, principal ideal domains, fields.
- (II) **Linear algebra** [Lan02, III.1–4, XVI]
  - modules, linear maps, submodules, quotient modules, direct sums,
  - multilinear maps, tensor products,
  - torsion, elementary divisor theorem for modules over PIDs.
- (III) **Homological algebra** [Wei93, I] (will be mostly done in the lecture)
  - exact sequences, 5-lemma, snake lemma,
  - projective and injective modules,
  - chain complexes.
- (IV) **Category theory** [Wei93, A.1–3, A.5]
  - categories, morphisms, functors, natural transformations,
  - universal objects: (co-)products, pullbacks, pushouts,
  - free and forgetful functors.

## REFERENCES

[Lan02] Serge Lang. *Algebra*. Springer, New York, 2002.

[Wei93] Charles Weibel. *An introduction to homological algebra*. Cambridge, 1993.