Knot theory

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Knots are embedded S^1's in $\mathbb{R}^3$. While $\pi_1(\mathbb{R}^3)$ is trivial, there are many non-trivial knots. In the seminar we will introduce different knot invariants and use these to detect and distinguish different knots. We will also consider the connection of knots and 3-dimensional manifolds.

Prerequisites:

Set theoretic topology and the fundamental group (as presented in "Einführung in die Geometrie und Topologie") as well as some basic algebra (as presented in "Einführung in die Algebra").

Talk 1: Knots and links

Definition of knots and links (give the PL- and the topological version); examples; existence of knot diagrams; Reidemeister moves; characterization of isotopy of knots by Reidemeister moves (without proof).
Literature: [BZ03, Section 1] and [Lic97, Section 1]

Talk 2: The knot group I

Definition of the knot group; Wirtinger presentation; computation of the knot group of the unknot and the trefoil; knot groups of torus knots.
Literature: [BZ03, Section 3.B] and [Rol90, Sections 3.B-3.D]

Talk 3: Homology

Definition and basic properties of singular homology.
Literature: e.g. [Lüc05]

Talk 4: The knot group II

Schoenflies theorem; Dehn's Lemma; Sphere and Loop theorem (all without proof); Asphericity of knot complements; the knot group detects the unknot.

Literature: [Lic97, Theorem 2.9 and section 11] and [Rol90, Sections 4.A and 4.B]

Talk 5: Seifert surfaces

Definition and existence of Seifert surfaces; examples; genus of a knot; additivity of genus; prime decomposition of knots.
Literature: [Rol90, Section 5] and [Lic97, Section 2]

Talk 6: Infinite cyclic covers

Alexander invariant; examples; the Alexander invariant from the knot group; examples again; additivity of the Alexander invariant.
Literature: [Rol90, Section 7.A, 7.B, 7.D and 7.E]

Talk 7: The Alexander polynomial

Linking numbers; the Seifert matrix; the Alexander polynomial.
Literature: [Rol90, Sections 5.D and 8.A-8.C]

Talk 8: The Jones polynomial

The Kauffman bracket; definition and properties of the Jones polynomial; examples.
Literature: [Lic97, Section 3] and [Kaw96, Section 8.1]

Talk 9: 3-manifolds and Heegaard splittings

Surgery; twist equivalence; definition and existence of Heegaard splitings; every closed connected orientable 3-manifold can be obtained by surgery on a link.
Literature: [Lic97, Section 12]

Talk 10: The signature and slice knots

Sylvester's law of inertia; signature of symmetric matrices and knots; additivity of the signature; definition of slice knots; examples; signature and Alexander polynomial for slice knots.
Literature: [Rol90, Section 8.E]

Talk 11: Concordance

Definition of concordance and the concordance group; the signature homomorphism; the matrix cobordism group.
Literature: [Rol90, Section 8.F] and [Kaw96, Section 12]

Talk 12: Braid groups

Definition of braids; representation of links by braids; braid groups and their presentation; Theorem of Artin.
Literature: [BZ03, Section 2.D and section 10.A up to 10.3]

Talk 13: Branched covers

Definition of branched covers; examples; branched covers over links; examples.
[Rol90, Sections 10.B-10.E]

References

[BZ03]   Gerhard Burde and Heiner Zieschang, *Knots. 2nd revised and extended ed.*, 2nd revised and extended ed. ed., Berlin: Walter de Gruyter, 2003 (English).

[Kaw96] Akio Kawauchi, *A survey of knot theory*, Basel: Birkhäuser, 1996 (English).

[Lic97] W.B.Raymond Lickorish, *An introduction to knot theory.*, New York, NY: Springer, 1997 (English).

[Lüc05] Wolfgang Lück, *Algebraische Topologie. Homologie und Mannigfaltigkeiten.*, Wiesbaden: Vieweg, 2005 (German).

[Rol90] Dale Rolfsen, *Knots and links. 2nd print. with corr.*, 2nd print. with corr. ed., Houston, TX: Publish or Perish, 1990 (English).