

Hopf Algebras

Prof. Dr. C.-F. Bödigheimer

Mondays 12:15 — 14:00 h, Seminar Room 1.007 (Begin: Monday, 1. April 2019)

(Organisational Meeting was Wednesday 30. January 2019.)

This seminar will be on Hopf algebras as they occur in topology. We start with necessary preliminaries on bi-algebras and co-algebras and will see the main examples for Hopf algebras; then we investigate some structure theorems and some applications.

There is an interesting survey article by P. Cartier [Ca] describing Hopf algebras in topology and group theory and applications in combinatorics.

The literature cited for each talk overlaps and sometimes only parts of the cited chapters will enter the talks.

Talks

- (1) **Algebras and Co-algebras** DIMITRIS OIKONOMOU (1.4.2019)
 Definition of algebras and co-algebras, multiplication and co-multiplication (diagonal). Examples. Graded versions. Tensor products, sub-algebras and sub-co-algebras, ideals and co-ideals, quotients. Hom-duals of algebras and co-algebras, the 'finite-dual' A° of an infinite-dimensional algebra A . Graded versions of everything.
 [Un, chapter 1], [Sw, chapter I+II+VI], [Ka, chapter II].
- (2) **Bi-Algebras** MANUEL HOFF (8.4.2019)
 Definition and examples of bi-algebras. Tensor products, direct sums. Ideals and quotients. Lie algebras, their envelopping algebras.
 [Un, chapter 2], [Sw, chapter III], [Ka, chapter II].
- (3) **Hopf Algebras** NILS WASSMUTH (15.4.2019)
 Antipode. Examples. Decomposable elements, group-like elements, primitive elements. Connectivity of graded Hopf algebras. Free Hopf algebras.
 [Sw, chapter III], [Un, chapter 3], [Ka, chapter II].
- (4) **Homology and Cohomology of Lie Groups** MICHAÏL ARABADJI (**Tue 23.4.2019 !!**)¹
 Basics: the existence of co-multiplications. Hopf structure for $H^*(G)$ and $H_*(G)$ and duality for Lie groups. Survey of homology and cohomology of Lie groups.
 [Ha, Chap. 3, Sect. 3.C].
- (5) **The Theorem of Hopf-Leray** URS FLOCK (29.4.2019)
 Connected graded commutative Hopf algebras over fields of characteristic zero are free.
 [Do, chap. VII, sect. 10], [Spa, chap. 5, sect. 8], [Bo].

¹Because of Easter Monday the seminar is this week moved to: **Tuesday, 23.4.2019, time 8:15 - 10:00, place seminar room N 0.003.**

- (6) **The Theorem of Borel** SIMON DIMPKER (6.5.2019)
 Connected graded commutative Hopf algebras over fields of positive characteristic.
 [Bo].
- (7) **Theorems of Cartier** TILL WEHRHAN (13.5.2019)
 Decomposition and more structure theorems.
 [Ca, 3.8]
- (8) **Differential Graded Hopf Algebras I** JENDRIK STELZNER (20.5.2019)
 Definition and examples, in particular the Chevalley-Eilenberg complex of a Lie algebra. Enveloping algebra. Homology of the primitive part $\text{Prim}(A)$ of a co-commutative graded Hopf algebra A over a field of characteristic zero.
 [Lo, appendix A], [Qui, appendix 2].
- (9) **Differential Graded Hopf Algebras II** TOBIAS FLECKENSTEIN (27.5.2019)
 Homology of the Lie algebra of matrices. Chevalley-Eilenberg complex. Cyclic homology. Theorem of Loday-Quillen-Tsygan (sketch of the proof).
 [Lo, chap. 10].
- (10) **Application to Homotopy Groups** MAXIMILIAN SCHIMPF (3.6.2019)
 Loop spaces of simply connected spaces. Hopf structure on $H_*(X; \mathbb{F})$ for a field of characteristic zero. Hurewicz map. Milnor-Moore Theorem: $\pi_*(X) \otimes \mathbb{F} \cong \text{Prim } H_*(X; \mathbb{F})$.
 [Mi-Mo], [Lo, appendix A].
- (11) **The Dual of the Steenrod Algebra** MIGUEL BARRERO (17.6.2019)
 Algebra of cohomology operations in mod-2 homology $H^*(X; \mathbb{F}_2)$. Diagonal and Hopf structure. Milnor basis. Theorem of Milnor-Moore.
 [Mi], [Mo-Ta, chap. 5, pp. 45-57].
- (12) **Quantum Groups** BASTIAAN CNOSSEN (1.7.2019)
 R-matrices and quasi-triangular structures. Quantum groups. Representations of braid groups. Markov's theorem and link invariants.
 [Un, chap. 4, sections 4.1 - 4.3], [Ka].
- (13) **Hopf Rings** JERONIMO GARCIA (8.7.2019)
 Definition and examples. Statement and sketch of the proof.
 [Ra-Wi].
- (14) **The cohomology of the symmetric groups as a Hopf ring**
 DOMENICO MARASCO + SIL LINSKENS (9.7.2019)
 Symmetric groups, their cohomology. Transfer as a second multiplication.
 [Gi-Sa-Si].

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- [Ca] P. Cartier: *A primer on Hopf algebras*. In: *Frontiers in Number Theory, Physics and Geometry*, volume 2, Springer Verlag (2007).
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- [Ha] A. Hatcher: *Algebraic Topology*. Cambridge University Press (2002).
- [Ka] C. Kassel: *Quantum Groups*. Springer Verlag (1995).
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