Winter School — **Moduli Spaces** — Schedule - Titles - Abstracts

time	Wednesday 2.1.08	Thursday 3.1.08	Friday 4.1.08	Saturday 5.1.08	Sunday 6.1.08
9:30 - 10:30		**Faber** (1)	**Faber** (2)	**Faber** (3)	
10:30 - 11:00		tea & coffee	tea & coffee	tea & coffee	
11:00 - 12:00		**Fantechi** (1)	**Bödigheimer** (3)	**Tillmann** (2)	Beethoven Walk
12:00 - 14:00	lunch (P)	lunch (P)	lunch (P)	lunch (N)	
14:00 - 15:00	registration tea & coffee	discussions	discussions	discussions	
15:00 - 16:00	**Manin** (1)	**Manin** (2)	**Fantechi** (2)	**Fantechi** (3)	
16:00 - 16:30	tea & coffee	tea & coffee	tea & coffee	tea & coffee	
16:30 - 17:30	**Bödigheimer** (1)	**Bödigheimer** (2)	**Tillmann** (1)	**Tillmann** (3)	
18:30		joint dinner			

<u>Abbreviations and explanations</u>

For the **titles and abstracts** of the mini courses see below.

All mini courses will be held in the **Kleiner Hörsaal** (Small Lecture Hall) of the **Mathematical Institute** of the University of Bonn in **Wegeler Strasse 10**, first floor.

Lunch : On Wednesday, Thursday and Friday we have made reservations in the Poppelsdorfer Mensa (Student Cafeteria), just a short walk from Wegelerstrasse.
On Wednesday, before the actual opening of the winter school, the meeting place is Wegeler Strasse 10.
On Saturday lunch is in the Nasse Mensa, a 15-min walk from Wegeler Strasse.

Joint Dinner : The dinner is at *Bönnsch*, a traditional brewery and restaurant, Sterntorbrücke 4, at 18:30 h, about 20 minutes from Wegelerstrasse (and close to the MPI).

Beethoven Walk : As a sightseeing event we have arranged for a guided walk through the inner city of Bonn, with Beethoven's birthplace as one of the high points. It starts at 11:00 h and takes about 90 minutes.

Titles

Carl-Friedrich Bödigheimer (University of Bonn) :
Hilbert uniformization of Riemann surfaces

Carel Faber (KTH Stockholm) :
Intersection theory of moduli spaces

Barbara Fantechi (SISSA Trieste) :
Virtual techniques

Yuri Manin (MPI Bonn) :
Modular spaces of curves with marked points and related operads

Ulrike Tillmann (Oxford) :
Moduli Spaces in Topology

Abstracts

Carl-Friedrich Bödigheimer

Hilbert Uniformization of Riemann Surfaces

Abstract :
This mini course is an introduction to a certain uniformization method for Riemann surfaces with boundary curves (or equivalently, with tangent vectors).

Let $\mathfrak{Mod}^m_{g,n}$ denote the moduli space of surfaces F of genus g with n boundary curves (or tangent vectors X_j at Q_j) and punctures P_i. Consider the space of pairs (F, u) where $[F] \in \mathfrak{Mod}^m_{g,n}$ and $u : F \to \mathbb{R}$ is a harmonic function with certain singularities at the punctures P_i and the points Q_j. This is a bundle $\mathfrak{Harm}^m_{g,n} \longrightarrow \mathfrak{Mod}^m_{g,n}$ with contractible fibre.

On the other hand we shall consider the space of parallel slit domains $\mathfrak{Par}^m_{g,n}$. For reasons of simplicity, take $n = 1$. Then a point is given by a grid of p horizontal and q vertical lines in the complex plane $\mathbb{C}$ together with permutations $\sigma_0, \ldots, \sigma_q \in \mathfrak{S}_{p+1}$; here $0 \le p \le 4g + 2m$ and $0 \le q \le 2g + m$, and the permutations must satisfy certain conditions reflecting the number of of punctures and the Euler characteristic of F. The permutation σ_k encodes the gluing of the rectangles in the k-th column of the grid.

The critical gradient flow lines and critical levels of function u give a unique decomposition of the surface F; they correspond to the horizontal resp. vertical lines of the grid. The Hilbert uniformization is now a homeomorphism $\mathfrak{H} : \mathfrak{Mod}^m_{g,n} \simeq \mathfrak{Harm}^m_{g,n} \longrightarrow \mathfrak{Par}^m_{g,n}$.

The interesting features of $\mathfrak{Par}^m_{g,n}$ are the following:
• The cell structure is very similar to the bar resolution of the symmetric groups.
• The complex $\mathfrak{Par}^m_{g,n}$ can be used to do explicit computations.
• The family of complexes $\mathfrak{Par}^m_{g,n}$ form an operad, and they are an algebra over the little cubes operad in $\mathbb{R}^2$.
• The concept can be generalized to surfaces in bordism categories, to non-orientable surfaces, and the like.

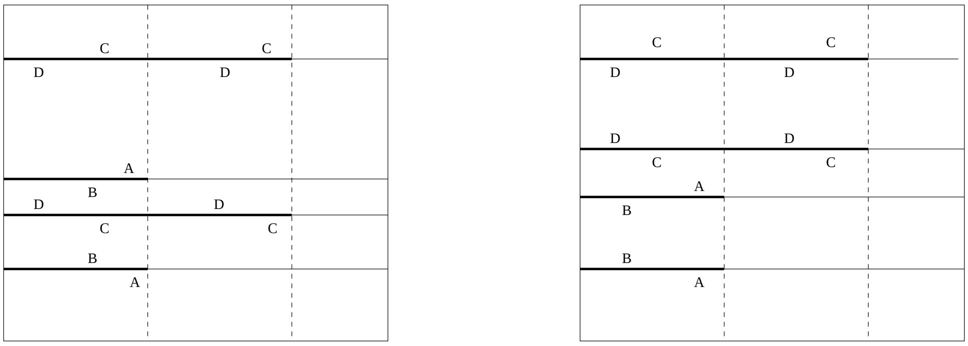

Figure 1: *Two examples of generic parallel slit domains with four slits: the letters indicate the gluing of the upper and lower banks of the rectangles. For the example on the left* $g = 1, m = 0, n = 1$*, and it represents the surface on the* <u>*conference poster*</u>*. See the following figures for the making of this surface. For the example on the right* $g = 0, m = 2, n = 1$*, and it is a twice punctured disc.*

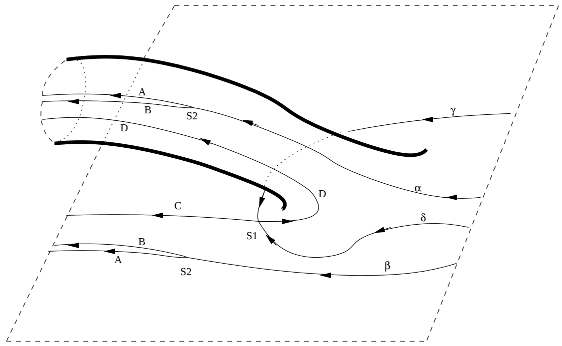

Figure 2: *The surface is half-way finished.*

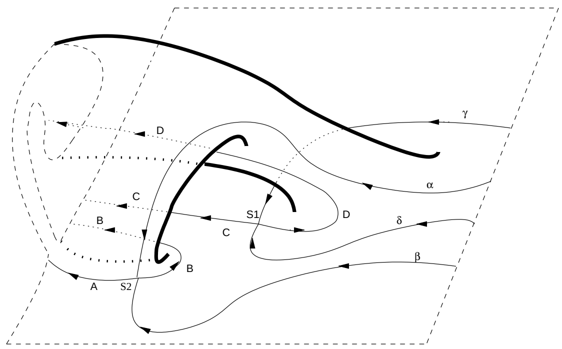

Figure 3: *This is the finished surface.*

The course uses Riemann surface theory (divisors and meromorphic functions), some Teichmüller theory (quasi-conformal maps), some topology (classifying spaces, simplicial sets, group cohomology) and some combinatorics.

Literature

[1] J.Abhau, C.-F. Bödigheimer, R.Ehrenfried: *Homology of the mapping class group* $\Gamma_{2,1}$ *for surfaces of genus* 2 *with*

boundary, to appear in Geometry and Topology Monographs, Festschrift for Heiner Zieschang.

[2] C.-F. Bödigheimer: *Configuration models for moduli spaces of Riemann surfaces with boundary*, Abh. Math. Sem. Univ. Hamburg 76 (2006), 191-233.

[3] C.-F. Bödigheimer: *Hilbert uniformization, I* (preprint).

[4] J.Ebert: *Hilbert uniformization, II* (preprint).

Carel Faber

Intersection theory of moduli spaces

Abstract :
In this minicourse, I will give an overview of known results and conjectures regarding this topic. Some of the main topics will be tautological classes and the relations between them, non-tautological classes, and the symmetric functions associated to cohomology groups of moduli spaces of pointed curves.

Barbara Fantechi

Virtual Techniques

Abstract:
After a brief outline of the definition of virtual class induced by a relative obstruction theory, we discuss some important results and outline applications for them: the pullback formula, Costello's pushforward formula, and Graber-Pandharipande virtual localization.

Yuri Manin

Modular spaces of curves with marked points and related operads

Abstract:
LECTURE 1. In this lecture, I will explain the general role of moduli spaces of curves with marked points as universal "cohomological operations" in algebraic geometry.

LECTURE 2. In this lecture, I will address the question on deformations and moduli in noncommutative geometry, in particular, based on operads.

Ulrike Tillmann

Moduli spaces in topology

Abstract:
About 25 years ago Mumford initiated the systematic study of the cohomology of Riemann surfaces. In this mini lecture course I will explain how a fusion of ideas from conformal field theory and algebraic topology led to the proof of Mumford's conjecture on the rational stable cohomology of moduli spaces [MW]. The methods of proof have been simplified and generalised [GMTW] yielding new applications in higher and lower dimensions.

Lecture 1. In my first lecture I plan to review the formalism of CFT and TFT, and in particular introduce a topological model of Segal's category of Riemann surfaces. I will then introduce the classifying space of a category in general and recall some of its basic properties and tools to compute its homotopy type.

Lecture 2. The second lecture will explain the main result of [GMTW]. This will include some revision of cobordism theory. Furthermore, I will explain
how the Mumford conjecture follows from these results and how torsion information can be extracted as well.

Lecture 3. In this last lecture I will present a sketch of the proof of the main result in [GMTW] and indicate how the results can be extended.

References

[MW] Ib Madsen, Michael Weiss, *The stable moduli space of Riemann surfaces: Mumford's conjecture*, Ann. Math. 2007, 843–941.

[GMTW] Soren Galatius, Ib Madsen, Michael Weiss, Ulrike Tillmann, Michael Weiss, *The homotopy type of the cobordism category*, http://uk.arxiv.org/pdf/math/0605249.pdf.