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Winter term 2016/17

Intersection theory and pure motives, Exercises – Week 9

Exercise 37. *Cohomology of decomposable motives.*
Fix a Weil cohomology theory H^* for k with coefficient field K and assume $\mathfrak{h}(X) \cong \bigoplus_{i=1}^n \mathbb{L}^{\otimes n_i}$ in $\mathrm{Mot}(k)$. Show that $H^{2i+1}(X) = 0$ for all i and $\sum \dim_K H^i(X) = n$.

Exercise 38. *Galois action on top cohomology.*
Let $k = \mathbb{F}_q$, $q = p^n$, and H^* a Weil cohomology theory. Show that for any geometrically integral $X \in \mathrm{SmProj}(k)$ the action of the Frobenius F^n on $H^{2d}(X)$ is given by $q^d \cdot \mathrm{id}$.

Exercise 39. *Adequate equivalence relation for (some) products of curves.*
Consider the adequate equivalence relations introduced in class (rational, numerical, homological, smash nilpotent) for the case of product of curves $C_1 \times C_2$. Start with $C \times \mathbb{P}^1$ and the square of an elliptic curve $E \times E$. In particular, in which cases do rational and numerical equivalence coincide? For which cases can you show that numerical and homological equivalence coincide?

Exercise 40. *Chords of twisted cubics.*
Let C, C' be two twisted cubics in $\mathbb{P}^3$ in general position (recall that a twisted cubic is the image of degree 3 Veronese embedding $\mathbb{P}^1 \hookrightarrow \mathbb{P}^3$). A *chord* of C is a line in $\mathbb{P}^3$ that meets C at two points. Taking for granted that the Chow ring of the Grassmannian $G(2,4)$ is generated by the cycles $\sigma_{1,0}$ and $\sigma_{1,1}$ (see exercise 26), compute the number of common chords of C and C'.

Exercise 41. *Cohomology of curves.*
Let C be a smooth projective curve of genus g. For an arbitrary Weil cohomology theory H^*, what is the dimension of $H^1(C)$?

Due Wednesday 21 December, 2016.