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Intersection theory and pure motives, Exercises – Week 3

Exercise 10. *Effective cycles*
Let X be a projective variety. Recall that a cycle $\alpha = \sum n_i[V_i]$ is called effective if $n_i \geq 0$ for all i. Prove that an effective cycle α on X is rationally equivalent to zero if and only if $\alpha = 0$.

Exercise 11. *Exterior product.*
Consider algebraic schemes X_1, X_2 over k, their product $X_1 \times X_2$, and the natural map $Z(X_1) \times Z(X_2) \to Z(X_1 \times X_2)$ given by extending linearly the map $([Y_1],[Y_2]) \mapsto [Y_1 \times Y_2]$. Show that this induces a bilinear map

$$\mathrm{CH}(X_1) \times \mathrm{CH}(X_2) \to \mathrm{CH}(X_1 \times X_2), \tag{1}$$

which is compatible with flat pull-back and proper push-forward. Compare this to the base change map $\mathrm{CH}(X) \to \mathrm{CH}(X_K)$ for finite field extensions K/k. Find an example where (1) is not surjective.

Exercise 12. *Minimality of rational equivalence.*
Assume that a subgroup $R(X) \subset Z(X)$ is given for all algebraic schemes X over a given field k. Assume that these subgroups are preserved under flat pull-back and proper push-forward and that $R(\mathbb{P}^1)$ contains $[p] - [q]$ where $p = [1 : 0]$, $q = [0 : 1]$. Show that then $\mathrm{Rat}(X) \subset R(X)$.

Exercise 13. *Zero-cycles of projective bundles.*
Let X be a variety over a field k (for simplicity you may assume that k is algebraically closed). Let E be a vector bundle on X. Prove that $\mathrm{CH}_0(\mathbb{P}(E)) \simeq \mathrm{CH}_0(X)$.

Further recommended exercises: Example 2.1.2 in Fulton's book.

Due Wednesday 9 November, 2016.