Research Statement

Sheila K. Miller

Left distributive algebras (LD's) are sets with one binary operation satisfying the law $a(bc) = (ab)(ac)$. The most common LD's are group conjugation and the weighted mean $(x * y = px + (1-p)y)$, each of which occur in the study of knots and braids. Beginning in the late 1980's, an interesting connection was discovered between free left distributive algebras and large cardinal axioms. While group conjugation and the weighted mean are left distributive, they are idempotent, hence not free. We denote the free left distributive algebra on one generator by $\mathcal{A}$ and the free LD on κ generators by $\mathcal{A}_\kappa$. $\mathcal{A}$ manifests fairly naturally in two contexts: as an algebra which exists under the assumption of the existence of a nontrivial rank embedding (a very strong large cardinal hypothesis), and as a particular operation on a subset of the braid group B_∞.

Define $p <_L q$ for $p, q \in \mathcal{A}$ if and only if q can be written as $(((pq_1)q_2)\cdots q_n)$, for some $q_1, q_2, \ldots q_n$. Laver [2] and Dehornoy [1] proved independently and by different methods that $<_L$ linearly orders $\mathcal{A}$. Laver's method demonstrated the linearity of $<_L$ by establishing the existence of a division form for the elements of $\mathcal{A}$, and this division form has consequences for the structure of $\mathcal{A}$. The division algorithm itself takes place in a larger algebra $\mathcal{P}$ that is formed by freely adding a composition operation, $\circ$, to $\mathcal{A}$.

The original proof of the division form theorem for $\mathcal{P}$ relies on the existence of another normal form theorem of Laver. I have given a new proof that establishes the result directly in the hopes that it will be more useful in generalizing the division form theorem to the many-generator case. There are significant complications that arise when considering terms in $\mathcal{A}_\kappa$ as opposed to the one generator case; as the κ distinct generators of $\mathcal{A}_\kappa$ are unordered by $<_L$, there is necessarily a new obstacle to comparing words. I hope to soon complete the generalization to achieve that every word in $\mathcal{P}_\kappa$ has a division form equivalent.

A related problem that I would be interested in pursuing at the workshop is that of demonstrating the existence of a copy of $\mathcal{A}_2$ from the assumption of the existence of a nontrivial embedding $j : V_\lambda \to V_\lambda$. I have considered a pair of embeddings generated from j such that I believe neither can be generated from applications of the left distributive law to the other, but I've not been able to prove it.

Most of the other problems I am working on (for example, various well-foundedness questions) are related to LD's, and are essentially algebraic in nature. I don't know that anyone else would be interested in them, though if they are I will happily discuss them!

References

[1] P. Dehornoy. Braid groups and left distributive structures. *Trans. Amer. Math. Soc*, 345, 1994.

[2] R. Laver. On the algebra of elementary embeddings of a rank into itself. *Advances in Math*, 110:334–346, 1995.