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My current work concerns the restrictions one has to put on the *Easton function*, or *continuum function*, on regular cardinals in the context of large cardinals with reflection properties (this would typically be measurable cardinals). More specifically, if  $F$  is an Easton function, i.e. for all regular cardinals  $\alpha, \beta$  we have  $\alpha < \beta \rightarrow F(\alpha) \leq F(\beta)$  and  $\text{cf}F(\alpha) > \alpha$ , we ask which cardinals  $\kappa$  remain measurable in a cofinality-preserving generic extension realizing  $F$ , i.e.  $2^\alpha = F(\alpha)$  for  $\alpha$  regular. The potential preservation of measurability of  $\kappa$  while the its power set has a prescribed value  $F(\kappa)$  allows for a subsequent singularization via a single Prikry sequence, obtaining a failure of SCH in the context of the given Easton function  $F$ .

By results of Gitik, if  $\kappa$  is measurable and  $2^\kappa = \lambda$ , we need at least the strength of  $o(\kappa) = \lambda$ , which is slightly weaker than  $\kappa$  being  $\lambda$ -hypermeasurable (this means that  $H(\lambda)$  of  $V$  is included in a target model of some elementary  $j : V \rightarrow M$  with critical point  $\kappa$ ). However, it seems that to obtain  $2^\kappa = \lambda$  while keeping  $\kappa$  measurable *and simultaneously* realizing an arbitrary  $F$  on all regular cardinals, we need the full strength of  $\lambda$ -hypermeasurability.

In particular, we have shown<sup>1</sup> that if  $F$  is an Easton function, then there is a cofinality-preserving generic extension  $V^*$  of  $V$  which preserves measurability of every  $\kappa$  satisfying the following single non-trivial condition:

- $\kappa$  is  $F(\kappa)$ -hypermeasurable in  $V$  and this is witnessed by an embedding  $j : V \rightarrow M$  such that  $j(F)(\kappa) \geq F(\kappa)$ .

Building on a work by Menas, we have also shown that if  $F$  is simply defined, then all strong cardinals are preserved in the generic extension  $V^*$ .

Future work might inquire whether one really needs the full strength of  $F(\kappa)$ -hypermeasurability in the above result, or what other large cardinals may be preserved. A more difficult question would be to what extent such results can be extended to Easton functions defined on singular cardinals as well.

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<sup>1</sup>Sy D. Friedman and Radek Honzik. Easton's theorem and large cardinals. Submitted to APAL.