Real forcing

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January 16, 2008

The first application of forcing was the consistency proof of ¬CH. The forcing notion that we now call "Cohen forcing" adds a large number (at least $\aleph_2$) of new reals without collapsing cardinals.

Since then, an almost uncountable number of forcing notions adding reals has been invented (Solovay=random, Sacks=perfect, Miller=superperfect, etc), plus a few methods (product, composition, iteration, amalgamation) of combining these forcing notions.

When designing a forcing notion to solve a specific problem, one usually has to take care of the following two aspects:

1. The forcing notion has to add a new object g (often a real number) satisfying some property X
   (as: g is faster, higher, stronger than all reals of the ground model).

2. The forcing notion should not add objects/reals r with some property Y
   (such as: r codes a well-order of ω of type ω_1^V, r destroys/trivialises this or that structure from the ground model).

In my tutorial I will give many examples for forcing notions adding reals, and explain why they add reals with some property X, and also (what is often more difficult) why they do not add reals with property Y. An important ingredient in such proofs are "preservation theorems", i.e., theorems of the form:

> Whenever forcing notions $P_1, P_2, \ldots$ are of a particularly nice form, then also the product/iteration/etc of these forcings has nice properties.