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My research concentrates on descriptive set theory, including the structure of the real line and properties of Borel functions.

In a joint paper with Janusz Pawlikowski (*Two stars*, to appear) we investigate an operation  $*$  on the subsets of  $\mathcal{P}(\mathbb{R})$  which is connected with Borel's strong measure zero sets as well as strongly meager. By definition,  $\mathcal{A}^* = \{B \in \mathcal{P}(\mathbb{R}) : \forall A \in \mathcal{A} A + B \neq \mathbb{R}\}$  for any  $\mathcal{A} \subseteq \mathcal{P}(\mathbb{R})$ . The results concern the behaviour of the family of countable sets when  $*$  is applied to it. We give a short proof of a theorem of Solecki stating that the family of countable subsets is a fixed point of  $**$  (superposition of  $*$  twice). We also construct a translation invariant  $\sigma$ -ideal  $I$  such that  $I^*$  is equal to the family of countable subsets of  $\mathbb{R}$ .

Recently, I was interested in the structure of Borel functions which are not  $\sigma$ -continuous (i.e. there is no countable family of subsets (arbitrary) of it's domain such that the function is continuous on each of these sets). It was an old open problem solved by Keldiš, Adyan and Novikov if such functions exist. However, a particularly simple example was given by Pawlikowski:  $P : (\omega + 1)^\omega \rightarrow (\omega)^\omega$  is defined as  $P(f)(n) = f(n) + 1 \pmod{\omega + 1}$ . It turns out (due to a theorem of Solecki) that this is actually the simplest such function (it factorises every Borel not  $\sigma$ -continuous function). I use the determinacy of Borel games to get some additional results in this field.