Set theory and model-theoretic logics

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Tutorial

One of the roles of logic is to serve as a tool for the study of structures. The best known tool, first order logic, cannot distinguish between cardinalities of infinite models. There are many extensions of first order logic where such and other sharper distinctions are possible. The most notable ones are the infinitary logics, logics with generalized quantifiers, and higher order logics. There are also intermediate logics which do not fit well into these three categories, such as the equicardinality quantifier "there are as many x with $\phi(x)$ as there are y with $\psi(y)$". It is an example of a *strong* logic, that is, a logic which has enough power to express properties of not only this or that model, but of the underlying set theoretical universe. The opposite is an *absolute* logic, that is, a logic the truth definition of which makes no reference to what kind of set there exists in the underlying universe.

In the tutorial I give an introduction to what is known about strong logics. What are strong logics, what are they good for, and how do they depend on set theoretical properties of the universe? What kind of compactness properties do they have? What kind of interpolation theorems? What kind of Löwenheim-Skolem properties? I will discuss in depth (1) the transfinite Ehrenfeucht-Fraïssé game, and (2) the equicardinality quantifier, both from a set theoretic point of view; methods, results and open problems.