

RESEARCH STATEMENT

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My PhD project is officially about partition properties *without the axiom of choice* (AC), but I like all large cardinals in ZF. At the moment I study higher Chang conjectures. Their consistency strength seems to drop considerably without AC. For example look at $(\omega_4, \omega_2) \rightarrow (\omega_2, \omega_1)$ which is inconsistent with choice. This Chang conjecture and all the others that ‘start’ with a successor of a regular cardinal, are not only consistent with ZF but equiconsistent with only one Erdős cardinal. In ZFC other higher Chang conjectures of that form have high consistency strength. For example, Donder and Koepke showed that if $(\kappa^{++}, \kappa^+) \rightarrow (\kappa^+, \kappa)$ and $\kappa \geq \omega_1$ then 0^\dagger exists. At the moment I’m working on a presentation of the consistency strengths of the Chang conjectures in ZF, as complete as possible.

Other large cardinals that become weaker or “small” without choice, are ones whose definition involves partitions and/or ultrafilters. Jech showed that ω_1 can be measurable and it’s easy to show that it can be also weakly compact, Ramsey and more. This indicates that having a normal measure is not a good definition for measurability and other large cardinals if you’re in a choiceless world. There, elementary embeddings should be used.

Measurable cardinals are indeed very interesting to me. Even the simple question of whether one can have a measurable with no normal measures doesn’t seem to have an obvious answer. I’m also puzzled with the problem of successive measurables; either having two or three in a row with assumptions below AD or having more than three successive measurables with *any* assumptions. This is a long standing open question. It apparently should be connected to Radin forcing and perhaps to Moti Gitik’s construction where every uncountable cardinal is singular. Measurable cardinals and these methods for working with them are things I’d like to get to know much better.

Equiconsistency proofs in the large cardinal realm are usually done with forcing for one side and core models for the other. In the forcing side I like using symmetric forcing which produces models without choice. I use Jech’s uniform method for symmetric forcing, only translated from Boolean valued models to forcing with partial orders. In the core model side, I have spent some time reading about the Dodd-Jensen core model. However, I’m happy to understand the basic ideas and just use black boxes from this very complicated theory. I do admire core model theory proofs and constructions but I prefer spending my time forcing. Therefore, core model theorists would be my main target for future collaboration.

Finally, another project I am working on with Peter Koepke is a paper on topological regularities in second order arithmetic (SOA). This is a project Peter Koepke had with Michael Möllerfeld. It is shown that ZFC is equiconsistent with SOA + “all sets of reals are Lebesgue measurable, have the Baire property and the perfect set property”. I helped this project in the forcing side, a class Lévy collapse of all the ordinals. This project will be continued by studying this model for further

topological regularities. Working on this has got me interested in class forcing, a very powerful method indeed.

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