RESEARCH STATEMENT

DAVID SCHRITTESSER

A set of reals (in $\mathbb{R}^n$, or more generally in any Polish space) is projective if it arises from an open (or Borel-) set (in $\mathbb{R}^k$) through repeatedly taking the complement or the image under projection maps. An example of a regularity property would be that of being Lebesgue-measurable, or that of having the property of Baire (these being the two examples studied in my thesis).

More generally, given a σ-ideal I on $\mathbb{R}$, we call a set of reals I-regular if and only if it is equal to a Borel set, modulo a set from I. Numerous examples of such ideals are well-studied, as they naturally arise in forcing theory of the reals.

It has long been known that we cannot, in ZFC, settle the question whether projective sets are Lebesgue-measurable (or have some other regularity property of your choice).

Two things are surprising in this field of study. Firstly, it should be possible to force every "simple" set (that is, up to a given level of the projective hierarchy) to be regular (in some specified sense), with regularity failing at the next level. Nonetheless, this has not been done yet from an optimal assumption, that is, the proof uses too strong hypotheses in the sense of assuming the consistency of relatively large cardinals. Secondly, until recently, all the known techniques to tamper with the regularity of sets in the projective hierarchy affected all the notions of regularity simultaneously. Yet in the general case it should not be expected that every set up to a certain level of complexity having regularity property A should mean that all of these sets have to be regular in a different sense, B.

The techniques developed in my thesis open up several possibilities dealing with questions such as these. My thesis solves the following case: we have a model where every projective set of reals is Lebesgue-measurable but there is a set without the Baire property, at the lowest level possible in the projective hierarchy. It remains to be seen if we can generalize this to be able to prescribe at which level non-regular sets occur. And it remains to be seen if we can do similar constructions with other notions of regularity. Both questions will not be solved by straightforward generalisations of the proof mentioned. The second question has to do with finding properties of ideals which, in a sense, allow to distinguish between the reals added by certain forcing notions.

This question is also of relevance to the intricate theory of what reals are added by a forcing. In fact, there are reasons to expect this line of research to yield interesting examples of adding reals, contributing to the general theory of forcing.

Another topic in descriptive set theory to which the techniques developed in my thesis may prove useful is the theory of so-called "small sets". I plan to investigate possible applications in this area.

Further research should also address the question whether we can adapt the forcing used to prove the result in my thesis in such a way as to allow for models where the continuum hypothesis fails; if it is the case that we have found a general approach to dealing with the regularity of projective sets, it should not rely on forcing the continuum to be $\aleph_1$ (as does the construction given in my thesis).

There is also no apparent reason that we should be confined to starting with L as a ground model. Being able to carry out the same construction starting from models incorporating large cardinals and then showing that some of their largeness is preserved would be a big step toward a complete understanding of regularity properties of projective sets; provided we can take it up to the level of large cardinals where "they take over" and imply regularity (up to some level), thus ruling out the kind of freedom our models would exploit.

Lastly, an important question left open in my thesis is the optimality of the large cardinal hypothesis in the proof. Although the hypothesis we use is very mild (the existence of a Mahlo cardinal), one would like to prove that this is the weakest possible hypothesis; this is not at all apparent. One scenario would be to try develop the theory of forcing iteration used in my thesis further, to show the proof actually works with a weaker assumption. But the other scenario - that the existence of a Mahlo is in fact optimal - is also feasible.

The general theory of iterated forcing is another field I plan to do research in. One of the questions I intend to address has to do with preservation theorems. A preservation theorem tells us that an iteration of forcing does not do unwanted damage if none of the iterands does. The known preservation theorems usually use small supports in the iteration, a harsh technical limitation. My thesis has an interesting example of an iteration which does no harm, yet none of the known iteration theorems apply. Moreover, this iteration uses an interesting kind of support. I plan to work on the question of finding a "nice" class of forcing, closed under iteration, which should include the forcing used in my thesis.

Another respect in which my future research can build on the iteration theory developed in my thesis is the question of which large cardinals are preserved in a forcing extension. In my thesis, I describe a class of forcing, dubbed stratified. This notion should be applicable when trying to retain large cardinal properties (e.g. measurability) when going to a forcing extension.