RESEARCH STATEMENT

Assaf Rinot (*Tel-Aviv University*),
Second year PhD student under the direction of M.Gitik.
`http://www.math.tau.ac.il/~rinot`

My primary research interest is *singular cardinals combinatorics*.

Background. Singular cardinals happens to have a non-trivial effect on usual mathematical objects. From one hand, being limit cardinals, the singular cardinals satisfy some plausible compactness properties, e.g., Shelah's theorem that every group of singular cardinality in which every subgroup of smaller cardinality is free - is itself free. On the other hand, being the limit of less than itself many smaller cardinals, it is often possible to carry out diagonalization arguments with respect to these cardinals, establishing implausible properties, e.g., Pouzet's theorem that every poset whose cofinality is a singular cardinal must contain an infinite antichain.

The most interesting case is whenever the question of satisfaction of a certain property of a singular cardinal is determined by its cardinal arithmetic configuration, e.g., as in [5]. The research of singular cardinals combinatorics centers at determining the exact interplay between different cardinal arithmetic configurations and related combinatorial properties.

Work so far. In their paper from 1981, Milner and Sauer conjectured that the following improvement of Pouzet's above-mentioned theorem should hold: if $\langle P, \leq\rangle$ is a poset whose cofinality is a singular cardinal λ, then P must contain an antichain of size $\mathrm{cf}(\lambda)$. Shortly afterwards, it has been observed by several authors including Hajnal, Prikry, Pouzet, and also Milner and Sauer that the conjecture is a consequence of GCH and some of its variants. However, to these days, a consistent counterexample is still unknown to exist.

In the last few years, a progress on this matter has been made in the form of an unpublished result of Magidor and independently the main result of [1], establishing that the Milner-Sauer conjecture has large cardinals consistency strength. Then, in [3], by pushing further the combinatorics of [2], it has been proved that this conjecture is a consequence of a certain, rather sharp, weakening of the GCH, whose consistency of its negation is a major open problem of modern cardinal arithmetic:

Definition (*Prevalent Singular Cardinals Hypothesis*). *For every singular cardinal λ, there exists a family $\mathcal{A} \subseteq \mathcal{P}(\lambda)$ of size λ with $\sup\{|A| \mid A \in \mathcal{A}\} < \lambda$ such that every $B \subseteq \lambda$ of size $< \mathrm{cf}(\lambda)$ is contained in some $A \in \mathcal{A}$.*

To describe another problem of a similar flavor, recall that given a topological space $\mathbb{X}$, its *density* describes the minimal (infinite) cardinality of a dense subspace, and its *weight* describes the minimal cardinality of a base for $\mathbb{X}$. By a recent exciting result of Juhász and Shelah, the existence of a regular hereditarily Lindelöf space of density $\aleph_{\omega_1}$ is consistent. Now, a diagonalization argument of [1] shows that such space must have more than $\aleph_{\omega_1}$ many open sets, but what about its weight? can it equal the density while preserving the hereditarily Lindelöfness?

In [4], it is proved that the existence of an hereditarily Lindelöf space of density and weight $\aleph_{\omega_1}$ entails the negation of the prevalent singular cardinals hypothesis.

Future plans. In order to determine whether the cardinal arithmetic configuration that was found to witness the negation of the Milner-Sauer conjecture (or the topological problem) indeed suffices to produce a counterexample, it is needed to find several ways to utilize negative cardinal arithmetic configurations in combinatorial constructions. An humble step in this direction appears in [5]. We shall also study Prikry-type forcing, specifically Gitik's forcing along short extenders, aiming at establishing the consistency of the cardinal arithmetic occurring in our combinatorial research, and trying to answer other well-known questions in the field of cardinal arithmetic and singular cardinals combinatorics.

References

[1] A. Rinot. **On the consistency strength of the Milner-Sauer conjecture.** *Ann. Pure Appl. Logic*, 140(1-3):110–119, 2006.

[2] A. Rinot. **Aspects of singular cofinality.** *Contrib. Discrete Math.*, 2(2):186–205, 2007.

[3] A. Rinot. **Antichains in partially ordered sets of singular cofinality.** *Arch. Math. Logic*, 46(5):457–464, 2007.

[4] A. Rinot. **On topological spaces of singular density and minimal weight.** *Topology Appl.*, 155(3):135–140, 2007.

[5] A. Rinot. **A topological reflection principle equivalent to Shelah's Strong Hypothesis.** *Proc. Amer. Math. Soc.*, accepted for publication, 2007.