

GRADUATE SEMINAR ON LOGIC (S4A4) WS 2021/22

O-minimality

Instructor. Philipp Hieronymi (phierony@illinois.edu)

Time and Place. Tuesdays 16 (ct) -18 (probably hybrid or on Zoom, possibly joint with graduate students at the University of Illinois)

Preliminary meeting. Wednesday July 28th 17 (ct)

<https://illinois.zoom.us/j/87963512953?pwd=MXBHOFdQZWcrMVYzaXRncmRvaXF0dz09>

Abstract. Originated in logic, o-minimality has recently seen many applications in number theory and geometry. This seminar serves as an introduction to the construction of o-minimal structures on the real field. The main results discussed in this seminar will be the o-minimality of the following structures:

- \mathbb{R}_{an} , the real field with all restricted analytic function ([1]),
- \mathbb{R}_{exp} , the real field with the exponential function ([6]), and
- $\mathbb{R}_{an,exp}$, the real field with the exponential function and all restricted analytic functions ([5, 4]).

While the consequences of o-minimality are outlined in [3], the book doesn't contain much about how to prove o-minimality of a given structure and postpones this to a further volume (which never appeared). This seminar covers some of the material of this second volume. Instead of following the original [1], we will read notes by Lou van Dries [2] (which you will receive by email) contain the proof of the o-minimality of \mathbb{R}_{an} . We will then go through [4] to prove o-minimality of $\mathbb{R}_{an,exp}$.

Prerequisites. Basic knowledge of first-order logic is definitely needed, but can be picked up easily. While prior knowledge of o-minimality might clarify the motivation why we want to construct o-minimal structures, it is not necessary to understand the content of this seminar. A solid background in algebra (in particular, basic valuation theory) is desirable.

Talks.

- (1) Formal power series, [2, p. 5-12]
- (2) Convergent power series and Weierstrass preparation, Flatness, [2, p. 13-22] (2 talks)
- (3) Analytic functions and Weierstrass preparation, [2, p. 23-30] (2 talks)
- (4) Quantifier elimination for \mathbb{R}_{an} , [2, p.30-36] (2 talks)
- (5) An axiomatization for T_{an} , the theory of \mathbb{R}_{an} , [4, Section 2]
- (6) Valuation theoretic properties of models of T_{an} , [4, Section 3]
- (7) The theory of (\mathbb{R}_{an}, \exp) , [4, Section 4]
- (8) O-minimality and Hardy fields, [4, Section 5]

REFERENCES

1. J. Denef and L. van den Dries, *p-adic and real subanalytic sets*, Ann. of Math. (2) **128** (1988), no. 1, 79–138. MR 951508
2. Lou van den Dries, *Notes for the next volume of tame topology and o-minimal structures*.
3. ———, *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series, vol. 248, Cambridge University Press, Cambridge, 1998. MR 1633348
4. Lou van den Dries, Angus Macintyre, and David Marker, *The elementary theory of restricted analytic fields with exponentiation*, Ann. of Math. (2) **140** (1994), no. 1, 183–205. MR 1289495
5. Lou van den Dries and Chris Miller, *On the real exponential field with restricted analytic functions*, Israel J. Math. **85** (1994), no. 1-3, 19–56. MR 1264338
6. A. J. Wilkie, *Model completeness results for expansions of the ordered field of real numbers by restricted Pfaffian functions and the exponential function*, J. Amer. Math. Soc. **9** (1996), no. 4, 1051–1094. MR 1398816