Models of Set Theory I. - Summer 2019

PD Dr. Philipp Lücke | Problem sheet 5

Problem 17. (1) (2 Points) Show that the canonical formula that defines the function $\mathsf{Sep} : \mathsf{Fml} \longrightarrow \mathsf{Fml}$ that sends an element of Fml to the canonical element of Fml that represents the corresponding instance of the Separation scheme is a Δ_1^{ST}-formula.

(2) (1 Points) Show that

$$\mathrm{ST} \vdash \forall k \in \mathsf{Fml}\ \forall M \neq \emptyset\ \forall 1 < n < \omega\ \forall a : n \longrightarrow M\ \big[\mathsf{Sat}(M, a, \mathsf{Sep}(k)) \\ \longleftrightarrow\ \exists x \in M\ \forall y \in M\ (y \in x\ \longleftrightarrow\ (y \in a(1) \wedge \mathsf{Sat}(M, a|_y^0, k)))\big]$$

(3) (2 Points) Show that the canonical formula that defines the function $\mathsf{Repl} : \mathsf{Fml} \longrightarrow \mathsf{Fml}$ that sends an element of Fml to the canonical element of Fml that represents the corresponding instance of the Replacement scheme is a Δ_1^{ST}-formula.

(4) (3 Points) Formulate and prove the analogue of (2) for the function Repl.

Problem 18 (4 Points)**.** Prove Lemma 1.3.19 from the lecture course: If κ is a strongly inaccessible cardinal, then $\mathsf{Sat}(V_\kappa, \emptyset, k)$ holds for every $k \in \ulcorner\mathrm{ZFC}\urcorner$.

Problem 19 (4 Points)**.** Prove Proposition 2.1.4 from the lecture course: There is a Δ_1^{ST}-formula $\varphi(v_0, v_1)$ with

$$\mathrm{ST} \vdash \text{"}\textit{The relation } \{(a,b) \mid \varphi(a,b)\} \textit{ is a well-ordering of the class } \mathrm{V}_\omega \textit{ of order-type } \omega\text{"}.$$

Problem 20 (4 Points)**.** Prove Lemma 2.1.5 from the lecture course: Assume ZF. If A is a class that is defined by an $\mathcal{L}_\in$-formula that only uses parameters from $A \cup \mathrm{Ord}$, then there is an $\mathcal{L}_\in$-formula $\Phi(v_0, v_1)$ with the property that for every $x \in \mathrm{OD}_A$, there is a function $a : n \longrightarrow A \cup \mathrm{Ord}$ with $n < \omega$ and $x = \{y \mid \Phi(y, a)\}$.

Please hand in your solutions on Monday, May 06, before the lecture.