Set Theory - Winter 2018/19

Prof. Dr. Peter Koepke, PD Dr. Philipp Lücke Problem sheet 5

Problem 19 (6 points)**.** Derive the following statements from the axioms of ZF:

(1) *Choice for finite collections*: Every finite set has a choice function.

(2) *The* Hausdorff Maximality Principle *for well-ordered partial orders*: For every partial order $(\mathbb{P}, \leq) \in V$ with the property that there exists a well-order $\prec$ on the underlying set $\mathbb{P}$, there is an inclusion-maximal chain X in $(\mathbb{P}, \leq)$.

Problem 20 (8 points)**.** Derive the following statements from the axioms of ZFC:

(1) Every vector space has a basis.

(2) *Existence of non-principle ultrafilters*: If X is an infinite set, then there exists $\mathcal{U} \subseteq \mathcal{P}(X)$ with the following properties:

  (a) If $x \in X$, then $\{x\} \notin \mathcal{U}$.

  (b) If $A, B \in \mathcal{U}$, then $A \cap B \in \mathcal{U}$.

  (c) If $A \in \mathcal{U}$ and $A \subseteq B \subseteq X$, then $B \in \mathcal{U}$.

  (d) If $A \subseteq X$, then either $A \in \mathcal{U}$ or $X \setminus A \in \mathcal{U}$.

Problem 21 (6 points)**.** Derive the following statements from the axioms of ZF:

(1) A finite union of finite sets is finite.

(2) A finite union of countable sets is countable.

(3) A countable union of finite sets of ordinals is countable.

Please hand in your solutions on Wednesday, November 14 before the lecture (Briefkästen 6 & 7).