Models of Set Theory II - Winter 2017/2018

Prof. Dr. Peter Koepke, Ana Njegomir — Problem sheet 12

Problem 1 (6 points)**.** Let $\mathbb{P}$ be a forcing notion. Prove that if $\mathbb{P}$ satisfies Axiom A, then it is proper.

Problem 2 (6 points)**.** The *Proper Forcing Axiom (PFA)* states that for every proper forcing $\mathbb{P}$, $\mathrm{FA}_{\aleph_1}(\mathbb{P})$ holds. Its consistency proof uses strong large cardinal assumptions. Show that if we replace $\aleph_1$ by $\aleph_2$ above, the statement becomes false.

Hint: Consider a forcing which collapses $\aleph_2$ to $\aleph_1$.

Problem 3 (6 points)**.** Let $\mathbb{P}$ be a forcing notion such that every uncountable subset of $\mathbb{P}$ contains an uncountable pairwise compatible set. Suppose that $\mathbb{T}$ is an $\aleph_1$-tree. Show that forcing with $\mathbb{P}$ adds no new uncountable branches through $\mathbb{T}$.

Problem 4 (4 points)**.** Let $\mathbb{U}$ denote the forcing $\langle [\omega]^\omega, \subseteq^*, \omega\rangle$. Prove that $\mathbb{U}$ is σ-closed.

Please hand in your solutions on Monday, January 15 before the lecture.