PROBLEM SET 8 - MODEL THEORY

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Problem 33 (4 points)**.** Read the *Appendix A* in "Katrin Tent & Martin Ziegler – *A course in model theory*" and the *Appendix A* in "David Marker – *Model Theory: An introduction*". Familiarize with the following concepts:

- Well-orderings, ordinals and cardinals.
- The $\aleph$-function.
- Regular and singular cardinals.
- The *Pigeonhole Principle.*
- Cardinalities of increasing sequences of sets.

Problem 34 (5 points)**.** Let $\mathcal{L}_T$ denote the first-order language consisting of unary predicate symbols P_s for every finite binary sequence $s \in {}^{<\omega}2$ and let TREE denote the $\mathcal{L}_T$-theory consisting of the following axioms:

- $\forall x \; P_\emptyset(x)$.
- $\exists x \; P_s(x)$ for every $s \in {}^{<\omega}2$.
- $\forall x \; [P_s(x) \longleftrightarrow (P_{s^\frown\langle 0\rangle}(x) \vee P_{s^\frown\langle 1\rangle}(x))]$ for every $s \in {}^{<\omega}2$.
- $\neg\exists x \; [P_{s^\frown\langle 0\rangle}(x) \wedge P_{s^\frown\langle 1\rangle}(x)]$ for every $s \in {}^{<\omega}2$.

Prove the following statements:

(1) TREE is complete.
(2) TREE has quantifier elimination.
(3) TREE is not totally transcendental.

Problem 35 (2 points)**.** Show that the theory of dense linear orders is not ω-stable.

Problem 36 (5 points)**.** (1) Prove that $\langle q \mid q \in \mathbb{Q}\rangle$ is a sequence of order indiscernibles for the model $\langle \mathbb{Q}, <\rangle$.
(2) Determine all sequences of order indiscernibles for the model $\langle \mathbb{Q}, <\rangle$ that are indexed by the natural numbers.

Problem 37 (4 points)**.** Use the *Ramsey Theorem* to prove the *Finite Ramsey Theorem*: Given $k, m, n \in \mathbb{N} \setminus \{0\}$, there exists an $N \in \mathbb{N}$ such that for every set X with at least N-many elements and every function $c : [X]^n \longrightarrow \{0, \ldots, k\}$, there is a c-homogeneous subset of X with at least m-many elements.

Please submit your solutions in the lecture on December 14.