PROBLEM SET 6 - MODEL THEORY

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Problem 20. (3 points) Prove that the the theory T_{RG} of the random graph has quantifier elimination.

Problem 21. (3 points) Suppose that κ is an infinite cardinal and $\mathbb{M} = (M, \dots)$ is a κ-saturated structure. Show that for every $n \geq 1$ every n-type over a subset A of M of size strictly less then κ is realized in $\mathbb{M}$.

Problem 22. (3 points) Let $\mathcal{L}$ be the language consisting of a single binary relation symbol E. Let T be the theory expressing that E is an equivalence relation, that all the equivalence classes are infinite and that there are infinitely many equivalence classes.

(a) For which infinite cardinals κ is T κ -categorical?
(b) Give a complete description of all $S_n(T)$.

Problem 23. (5 points) Suppose that T is a theory such that for every $n \geq 1$ every n-type $t(\vec{x}) = t(x_0, \dots, x_{n-1})$ follows from the set of quantifier-free formulas in $t(\vec{x})$. Show that T has quantifier elimination. *Hint: you can write any basic open set* $[\varphi(\vec{x})]$ *as a union of sets of the form* $[\psi(\vec{x})]$ *with* $\psi(\vec{x})$ *quantifier-free and apply compactness, or alternatively use the compactness theorem.*

Problem 24. (3 points) Let $\mathcal{L} = \{<, c_0, c_1, \dots\}$ and T_0 the theory of dense linear orders without end points. Find a complete $\mathcal{L}$-theory T extending T_0 that has 2^ω many 1-types and no countable saturated model.

Problem 25. (3 points) Let K_n the complete graph with n vertices. For which $n \geq 2$ is the class of finite K_n-free graphs an amalgamation class?

Please submit your solutions in the lecture on December 1.