PROBLEM SET 4 - MODEL THEORY

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Problem 15. (3 points) Show that any two $\mathcal{L}$-structures $\mathbb{A}$ and $\mathbb{B}$ are elementarily equivalent if and only if there are elementary embeddings $f\colon \mathbb{A} \to \mathbb{C}$ and $g\colon \mathbb{B} \to \mathbb{C}$ for some $\mathcal{L}$-structure $\mathbb{C}$.

Problem 16. (5 points) Show that every injective polynomial map $f\colon \mathbb{C}^n \to \mathbb{C}^n$ is surjective. *Hint: prove this first for finite fields and then for the algebraic closure $\bar{\mathbb{F}}_p$ of $\mathbb{F}_p$ for any prime p by working with finite subfields of $\bar{\mathbb{F}}_p$.*

Problem 17. (4 points) Decide which of the following classes is an amalgamation class.

(a) The class of finite triangle-free graphs.
(b) The class of finite acyclic graphs (i.e. graphs without cycles).

Problem 18. (5 points) Prove the following statements for any prime p, assuming that $\mathbb{F}_{p^n}$ is a subset of a fixed algebraic closure K of $\mathbb{F}_p$ for all $n \geq 1$.

(a) $\mathbb{F}_{p^m}$ is a subset of $\mathbb{F}_{p^n}$ if and only if $m|n$ (i.e. m divides n). *Hint: use the fact that $\mathbb{F}_{p^l}$ is the set of roots of the polynomial $X^{p^l} - X$ for all $l \geq 1$.*
(b) Give a direct proof that the class of finite fields with characteristic p is an amalgamation class.

Problem 19. (3 points) Show that the theory of infinite structures in the empty language $L_\emptyset$ has quantifier elimination.

Please submit your solutions in the lecture on November 16.