Set theory - Winter semester 2016-17

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Problem 14 (5 Punkte)**.** Suppose that $B \subseteq A$ and $f\colon A \to B$ is injective. In ZF, prove that there is a bijection $g\colon A \to B$ as follows.

(1) Let $A_0 = A$, $A_{n+1} = f[A_n]$, $B_0 = B$, $B_{n+1} = f[B_n]$ for $n \in \omega$ by recursion.

(2) Let

$$g(x) = \begin{cases} f(x) & \text{if } x \in A_n \setminus B_n \text{ for some } n, \\ x & \text{otherwise.} \end{cases}$$

Show that $g\colon A \to B$ is bijective.

Problem 15 (6 points)**.** The *Axiom of Dependent Choice* DC is the following statement:

Suppose that R is a (binary) relation on a set a with $dom(R) = a$. Then for every $x \in a$, there is a sequence $(x_n)_{n\in\omega}$ such that $x_0 = x$, $x_n \in a$, and $x_n R x_{n+1}$ for all n.

Prove the following statements in ZF.

(1) The Axiom of Choice AC implies DC.

(2) Suppose that DC holds and $\prec$ is a (binary) relation on a set b. Then $(b, \prec)$ is wellfounded if and only if there is no sequence $(x_n)_{n\in\omega}$ with $x_{n+1} \prec x_n$ for all n.

Problem 16 (4 Punkte)**.** Prove that the following statements are equivalent.

(1) AC

(2) Every surjective function $f\colon x \to y$ has a left inverse, i.e. a function $g\colon y \to x$ with $g \circ f = \mathrm{id}_y$.

Problem 17 (3 points)**.** Prove the following finitary choice principle in ZF without the axiom of choice: If $F : n \to V$ is a function such that $F(i) \neq \emptyset$ for all $i < n$, then there is $f : n \to V$ such that $f(i) \in F(i)$ for all $i < n$.

Due Friday, November 25, 2016, before the lecture.