Set theory - Winter semester 2016-17

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Problem 10 (4 points)**.** Prove the following statements.

(1) Ord is a proper class.

(2) There is no function $f\colon \omega \to \mathrm{Ord}$ such that $f(n+1) < f(n)$ for all $n \in \omega$.

Problem 11 (6 points)**.** Prove the following principles of $\in$-*induction* and $\in$-*recursion*.

(1) Let $\varphi(x, v_0, \ldots, x_{n-1})$ be an $\in$-formula and $x_0, \ldots, x_{n-1} \in V$. Suppose that

$$\forall x[(\forall y \in x\ \varphi(y, x_0, \ldots, x_{n-1})) \to \varphi(x, x_0, \ldots, x_{n-1})]$$

holds. Then $\forall x\ \varphi(x, x_0, \ldots, x_{n-1})$.

(2) Let $G\colon V \to V$. Then there is a canonical class term F such that $F\colon V \to V$ and $\forall x\ F(x) = G(F{\restriction}x)$, and moreover $F = F'$ for any class term F' with this property.

Problem 12 (8 points)**.** Prove the following statements for all ordinals α, β, γ.

(1) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$.

(2) $(\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$.

(3) $\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$.

(4) $+$ and $\cdot$ are not commutative on *Ord*.

Problem 13 (4 points)**.** (1) Suppose that α, γ are ordinals with $\alpha \leq \gamma$. Prove that there is a unique ordinal β such that $\alpha + \beta = \gamma$.

(2) Suppose that α, δ are ordinals with $\alpha \leq \delta$. Prove that there are unique ordinals β, γ such that $\alpha \cdot \beta + \gamma = \delta$ and $\gamma < \alpha$.

Due Friday, November 11, before the lecture.