Set theory - Winter semester 2016-17

Problems — Series 1

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Problem 1 (4 points)**.** Prove the following statements.

(1) $\bigcup V = V$.
(2) $\bigcap V = \emptyset$.
(3) $\bigcup \emptyset = \emptyset$.
(4) $\bigcap \emptyset = V$.

Problem 2 (4 points)**.** Suppose that F, G are functions.

(1) Show that $F = G$ if and only if $\text{dom}(F) = \text{dom}(G)$ and $F(x) = G(x)$ for all $x \in \text{dom}(F) = \text{dom}(G)$,
(2) Show that F is injective if and only if there is a function H with $\text{dom}(H) = \text{ran}(F)$ and $H(F(x)) = x$ for all $x \in \text{dom}(F)$.

Problem 3 (4 points)**.**
(1) Show that $\langle x, y\rangle := \{\{x, 0\}, \{y, \{y, \emptyset\}\}\}$ also satisfies the fundamental property of ordered pairs.
(2) Can $\{x, \{y, \emptyset\}\}$ be used as an ordered pair?

Problem 4 (4 points)**.** Prove the following statements.

(1) $\forall x \forall y \exists z\ z = (x, y)$.
(2) If $(x, y) \in A$, then $x, y \in \bigcup\bigcup A$.

Problem 5 (4 points)**.** Define a relation $\sim$ on V by $x \sim y \leftrightarrow$ *there is a bijective function* $f : x \to y$. One says that x and y are *equinumerous* or *equipollent.* Show that $\sim$ is an equivalence relation on V. What is the equivalence class of $\emptyset$? What is the equivalence class of $\{\emptyset\}$

Due Friday, October 28, before the lecture.