Set theory - Winter Semester 2014

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Problem 10 (3 points)**.** Prove in ZF^- that $a \times b = \{(x, y) \mid x \in a \wedge y \in b\}$ is a set, for all sets a, b.

Problem 11 (3 points)**.** Prove the following transfinite induction principle: Let $\varphi(x) = \varphi(x, v_0, ..., v_{n-1})$ be an $\in$-formula and $\bar{x} = (x_0, ..., x_{n-1}) \in V$. Assume

(a) $\varphi(0, \bar{x})$ (the initial case),

(b) $\forall \alpha \in Ord\ (\varphi(\alpha, \bar{x}) \to \varphi(\alpha + 1, \bar{x}))$ (the successor step),

(c) $\forall \lambda \in Lim\ (\forall \alpha < \lambda\ \varphi(\alpha, \bar{x}) \to \varphi(\lambda, \bar{x}))$ (the limit step).

Then $\forall \alpha \in Ord\ \varphi(\alpha, \bar{x})$.

Problem 12 (10 points)**.** Prove the following statements.

(1) If $x \subseteq Card$ is a set, then $sup(x) = \bigcup x$ is a cardinal.

(2) Every infinite cardinal is a limit ordinal.

(3) $Card$ is a proper class.

(4) For every infinite cardinal κ, there is an ordinal α with $\kappa = \aleph_\alpha$.

Problem 13 (6 points)**.** Prove that cardinal arithmetic is equal to ordinal arithmetic on ω, i.e. for all $m, n \in \omega$

(1) $m + n = m +_{ord} n$,

(2) $m \cdot n = m \cdot_{ord} n$.

Due Wednesday, October 29, before the lecture, in the mailboxes 6 and 7 for your tutorial, on the ground floor of the math department, Endenicher Allee 60.