Set theory - Winter Semester 2014

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Problem 5 (3 points)**.** The axiom system consisting of ZF^- together with the Power Set Axiom is called ZF. Show that ZF implies the Collection Scheme defined in Problem 4. *(Hint: Use the Von Neumann hierarchy.)*

Problem 6 (10 points)**.** Prove the following statements for all ordinals α, β, γ.

(1) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$.
(2) $(\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$.
(3) $\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$.
(4) $+$ and $\cdot$ are not commutative on Ord.

Problem 7 (6 points)**.**

(1) Show that the function

$$f\colon V \longrightarrow V;\ x \mapsto x \cup \{x\}$$

is injective.

(2) Show that $+$ and $\cdot$ are commutative on ω.

By Problems 6+7 and the lecture, the structure $\mathbf{N} = (\omega, 0, 1, +, \cdot)$ satisfies the axioms of Peano arithmetic. In particular, the structure $\mathbf{N}$ has the usual properties of the natural numbers.

Problem 8 (4 points)**.** Show that the structure $\mathbf{Q} = (\mathbb{Q}, 0, 1, +, \cdot)$ (as defined in the lecture) is a field of characteristic 0 with no proper subfield. You can assume that $+$ and $\cdot$ are associative, commutative and distributive.

Problem 9 (3 points)**.** Show that for all $x \in V$, $\mathrm{rank}(x) = \alpha$ if and only if α is least with $x \in V_{\alpha+1}$.

Due Wednesday, October 22, before the lecture, in the mailboxes 6 and 7 for your tutorial, on the ground floor of the math department, Endenicher Allee 60.