Set theory - Winter semester 2014

Problems	Dr. Philipp Schlicht
Series 1	Dr. Philipp Lücke

In the first three exercises, work in the system ZF^-.

Problem 1 (9 points)**.** Prove the following statements.

(1) $\forall x(x \neq \emptyset \rightarrow \bigcap x \in V)$.

(2) $\{\{x\} \mid x \in V\} \notin V$.

(3) A set z is *transitive* if $x \in z$ holds for all $y \in z$ and all $x \in y$. Prove that $\emptyset \in x$ holds for every transitive set $x \neq \emptyset$.

Problem 2 (6 points)**.**

(1) Show that $\langle x, y\rangle := \{\{x, \emptyset\}, \{y, \{\{\emptyset\}\}\}\}$ satisfies the fundamental property of ordered pairs.

(2) Does $\langle x, y\rangle := \{x, \{y, \emptyset\}\}$ satisfy the fundamental property of ordered pairs?

Problem 3 (6 points)**.** Suppose that F, G are functions.

(1) Show that $F = G$ if and only if $\mathrm{dom}(F) = \mathrm{dom}(G)$ and $F(x) = G(x)$ for all $x \in \mathrm{dom}(F) = \mathrm{dom}(G)$,

(2) Show that F is injective if and only if there is a function H with $\mathrm{dom}(H) = \mathrm{ran}(F)$ and $H(F(x)) = x$ for all $x \in \mathrm{dom}(F)$.

Problem 4 (3 points)**.** The *Collection Scheme* states that for every relation R and every set x, there is a set y such that for every $u \in x$, if there is some v with uRv, then there is some $v \in y$ with uRv. Prove that the axioms and schemes of ZF^- without the Replacement Scheme with the Collection Scheme imply the Replacement Scheme.

Due Wednesday, October 15, before the lecture, in the mailboxes 6 and 7 for your tutorial, on the ground floor of the math department, Endenicher Allee 60.