

CONTINUOUS IMAGES OF CLOSED SETS IN GENERALIZED BAIRE SPACES

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ABSTRACT. Let κ be an uncountable cardinal with $\kappa = \kappa^{<\kappa}$. Given a cardinal μ , we equip the set ${}^\kappa\mu$ consisting of all functions from κ to μ with the topology whose basic open sets consist of all extensions of partial functions of cardinality less than κ . We prove results that allow us to separate several classes of subsets of ${}^\kappa\kappa$ that consist of continuous images of closed subsets of spaces of the form ${}^\kappa\mu$. Important examples of such results are the following: (i) there is a closed subset of ${}^\kappa\kappa$ that is not a continuous image of ${}^\kappa\kappa$; (ii) there is an injective continuous image of ${}^\kappa\kappa$ that is not κ -Borel (i.e. that is not contained in the smallest algebra of sets on ${}^\kappa\kappa$ that contains all open subsets and is closed under κ -unions); (iii) the statement “*every continuous image of ${}^\kappa\kappa$ is an injective continuous image of a closed subset of ${}^\kappa\kappa$* ” is independent of the axioms of ZFC; and (iv) the axioms of ZFC do not prove that the assumption “ $2^\kappa > \kappa^+$ ” implies the statement “*every closed subset of ${}^\kappa\kappa$ is a continuous image of ${}^\kappa(\kappa^+)$* ” or its negation.

This is joint work with Philipp Schlicht.

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