Models of Set Theory II - Winter 2013

Prof. Dr. Peter Koepke, Dr. Philipp Schlicht Problem sheet 10

Problem 36 (6 Points)**.** Suppose that M is a ground model and $x \in {}^{\omega}2$. Show that x is Cohen generic over M if and only if $x \notin d^{M[G]}$ for all F-codes $d \in M$ for closed nowhere dense subsets of ${}^{\omega}2$.

Problem 37 (6 Points)**.** Let $(x+y)(n) = x(n) + y(n) \pmod 2$ for $x, y \in {}^{\omega}2$. Let $A + B = \{x + y \mid x \in A,\ y \in B\}$ for $A, B \subseteq {}^{\omega}2$. Suppose that $A, B \subseteq {}^{\omega}2$ are Borel sets of positive measure. Show that $A + B$ contains an interval.
(Hint: there are $m \in \omega$ and $s, t \in {}^{m}2$ with $\frac{\mu(A \cap N_s)}{\mu(N_s)} > \frac{1}{2}$ and $\frac{\mu(B \cap N_t)}{\mu(N_t)} > \frac{1}{2}$ by the Lebesgue density theorem. Show that $N_{s+t} \subseteq A + B$. To do this, show that $(z + (A \cap N_s)) \cap (B \cap N_t) = \neq \emptyset$.)

Problem 38 (6 Points)**.** Let P denote the forcing consisting of conditions $p = (C_p, F_p)$ with $dom(C_p) = n \in \omega$, $C_p \colon n \to [\omega]^{<\omega}$, $|C_p(m)| \leq 2^m$ for all $m < n$, and $F_p \in [{}^{\omega}\omega]^{\leq 2^n}$. Suppose that $p = (C_p, F_p)$, $q = (C_q, F_q)$. Let $p \leq q :\Longleftrightarrow C_q \subseteq C_p$, $F_q \subseteq F_p$, and $f(n) \in C_p(n)$ for all $n \in dom(C_p) \setminus dom(C_q)$ and all $f \in F_q$.

(a) Show that P is σ-linked.
(*Hint: suppose that $p = (C_p, F_p)$ and $q = (C_q, F_q)$. Show that p, q are compatible if $|\{f(n) \mid f \in F_p \cup F_q\}| \leq 2^n$.)*

(b) A 2^n-*cone* is a function $f \colon \omega \to [\omega]^{<\omega}$ with $|f(n)| \leq 2^n$ for all $n \in \omega$. Let $[f] := \{g \in {}^{\omega}\omega \mid \forall n \in \omega\ g(n) \in f(n)$. Suppose that M is a ground model and G is an M-generic filter for P^M. Show that in $M[G]$, there is a sequence $(f_m)_{m\in\omega}$ of 2^n-cones such that $({}^{\omega}\omega)^M \subseteq \bigcup_{m\in\omega}[f_m]$.

Please hand in your solutions on Monday, January 13 before the lecture.