SPECIALIZING ARONSZAJN TREES AND SQUARE SEQUENCES BY FORCING

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ABSTRACT. Let κ be an infinite cardinal and T be a κ^+-Aronszajn tree. Then it is possible to specialize T by forcing with the $<\kappa$-closed partial order $\mathbb{P}_T$ consisting of partial specializing functions of cardinality less than κ. If T has height ω_1, then a result of Baumgartner, Malitz and Reinhardt says that the partial order $\mathbb{P}_T$ satisfies the countable chain condition. In contrast, Laver's construction of κ^+-Aronszajn trees with ascent paths shows that forcings with $\mathbb{P}_T$ can collapse κ^+ if κ is an uncountable cardinal.

Given an uncountable cardinal κ with $\kappa = \kappa^{<\kappa}$, we will show that the existence of antichains of size κ^+ in $\mathbb{P}_T$ is equivalent to a certain combinatorial property of the tree T. We will then show that the canonical κ^+-Aronszajn tree $T(\rho_0)$ constructed by Todorčević from a $\square(\kappa^+)$-sequence does not satisfy this property and therefore can be specialized by a $<\kappa$-closed partial order satisfying the κ^+-chain condition. This observation can be used to derive lower bounds for the consistency strength of generalizations of Martin's Axiom to certain classes of $<\kappa$-closed partial orders satisfying the κ^+-chain condition whose consistency can be established using large cardinals.

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