On compactness properties of successors of singular cardinals

Menachem Magidor//Hebrew University of Jerusalem

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A cardinal κ is said to be compact for a given (typically second order) property if every structure of cardinality κ having the property also has a substructure of smaller cardinality having the property.

For a given property, finding which cardinals can be compact with respect to the given property leads to some very interesting set theoretical combinatorics. This is especially true when one tries to get a successor of a singular cardinal to be compact with respect to the given property. Typical properties which generates some interesting Set Theory are for instance the property of a (Abelian) group being free, a collection of sets having a one to one choice function, a graph having an uncountable chromatic number In some sense etc. In some sense problems like stationary sets reflections, tree property, the singular cardinals hypothesis etc. can be considered to be problems of the same type.

In the talk we shall survey some consistency results claiming that "small" cardinals like $\aleph_{\omega+1}$ or $\aleph_{\omega^2+1}$ can be compact with respect to interesting properties. We shall especially be interested in potential implications between compactness properties of successors of singulars. e.g. the tree property and stationary set reflection etc.