Models of Set Theory I - Summer 2013

Prof. Dr. Peter Koepke, Dr. Philipp Lücke Problem sheet 2

Problem 5 (4 Points)**.** Assume ZFC and define $W = V \setminus V_\omega$. Examine which ZFC-axioms hold in W.

Problem 6 (4 Points)**.** Assume ZF.

(1) Let π be an $\in$-automorphism of V, i.e. $\pi : V \longrightarrow V$ is a bijective class-function such that both π and π^{-1} preserve the $\in$-relation. Prove that π is the identity.

(2) Construct an $\in$-automorphism of $V \setminus V_\omega$ that is not the identity (Hint: *Consider a suitable permutation of* $V_{\omega+1} \setminus V_\omega$.).

Problem 7 (4 Points)**.** Assume ZF. Show that there is a well-ordering of V_ω that is definable by an $\in$-formula $\varphi(v_0, v_1)$ without parameters.

Problem 8 (4 Points)**.** Assume ZF. Let F be a finite set of formulas closed under subformulas. Define C_F to be the class of all ordinals α with the property that all formulas in F are V_α-V-absolute. Show that C_F is a closed and unbounded subclass of Ord.

Problem 9 (4 Points)**.** Assume ZF. Let $\varphi(v_0, \dots, v_{n-1})$ be an $\in$-formula. Then

$$\forall v_0, \dots, v_{n-1} \in M \ \big((M, \in) \models \ulcorner \varphi \urcorner [v_0, \dots, v_{n-1}] \ \longleftrightarrow \ \varphi^M\big)$$

holds for every set M.

Please hand in your solutions on Wednesday, April 24 before the lecture.