4. Problem sheet for Set Theory, Winter 2012

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Problem 11. Let A be a class term and suppose that that all $x \in A$ are transitive. Show that $\bigcup A$ and $\bigcap A$ are transitive.

Problem 12. Prove:

a) Let $A \subseteq Ord$ be a class term, $A \neq \emptyset$. Then $\bigcap A \in Ord$.

b) Let $x \subseteq Ord$ be a set. Then $\bigcup x \in Ord$.

Problem 13. Prove the following transfinite induction principle: Let $\varphi(x) = \varphi(x, v_0, ..., v_{n-1})$ be an $\in$-formula and $\bar{x} = (x_0, ..., x_{n-1}) \in V$. Assume

a) $\varphi(0, \bar{x})$ (the initial case),

b) $\forall \alpha \in Ord\ (\varphi(\alpha, \bar{x}) \to \varphi(\alpha + 1, \bar{x}))$ (the successor step),

c) $\forall \lambda \in Lim\ (\forall \alpha < \lambda\ \varphi(\alpha, \bar{x}) \to \varphi(\lambda, \bar{x}))$ (the limit step).

Then $\forall \alpha \in Ord\ \varphi(\alpha, \bar{x})$.

Problem 14.

(1) Suppose $\gamma \in Ord$. Show that $\bigcup(\gamma + 1) = \gamma$ and $Lim(\gamma) \to \bigcup \gamma = \gamma$.

(2) Show that $n \in \omega \leftrightarrow (n = 0 \vee \exists m \in n\ (n = m + 1)) \wedge \forall m \in n (m = 0 \vee \exists l \in m\ (m = l + 1))$.

There are 6 points for each problem. Please hand in your solutions on Monday, November 5 before the lecture.