3. Problem sheet for Set Theory, Winter 2012

Prof. Dr. Peter Koepke, Dr. Philipp Schlicht Mathematisches Institut, Universität Bonn, 22.10.2012

Problem 7. (Partial orders) Suppose R is a binary relation on a class X. Let $id_X = \{(x, x) \mid x \in X\}$. Show

- a) If R is a partial order on X, then $R \setminus id_X$ is a strict partial order on X.
- b) If R is a strict partial order on X, then $R \cup id_X$ is a partial order on X.

Problem 8. (Set existence and products) Prove

- a) The Set Existence Axiom from the other axioms.
- b) $\forall x \forall y \exists z \ z = x \times y$ without using the Power Set Axiom.

State which axioms you use.

Problem 9. (Successor function) Prove

- a) $\forall x \ x \neq x+1$.
- b) The successor function $s: V \to V, s(x) = x + 1$ is injective.

Problem 10. (Induction) Formulate a principle for R-induction on a class D and prove that this is equivalent to the assumption that R is a well-founded relation on D.

There are 6 points for each problem. Please hand in your solutions on Monday, October 29 before the lecture.