# THE EMBEDDABILITY RELATION ON MODELS OF SIZE $\kappa$ IS (STRONGLY) INVARIANTLY UNIVERSAL WHEN $\kappa^{<\kappa} = \kappa$.

LUCA MOTTO ROS

Abstract. Given an uncountable cardinal $\kappa$, we analyze the descriptive set-theoretical complexity of the embeddability relation on models of size $\kappa$ of a given $\mathcal{L}_{\kappa^+\kappa}$-sentence. In particular, we show that if $\kappa^{<\kappa} = \kappa$, then the relation $\sqsubseteq_\kappa$ of embeddability on models of size $\kappa$ is strongly invariantly universal for analytic quasi-orders, that is: for every analytic quasi-order $R$ on the generalized Cantor space ${}^\kappa 2$ there is an $\mathcal{L}_{\kappa^+\kappa}$-sentence $\psi$ such that the restriction of $\sqsubseteq_\kappa$ to the models of $\psi$ is Borel-isomorphic to $R$. As a consequence, we get that such embeddability relations fully characterize the class of all analytic quasi-orders on arbitrary standard Borel $\kappa$-spaces. This work extends previous results dealing with the special cases $\kappa = \omega$ (Friedman-Motto Ros 2010) and $\kappa$ weakly compact (Motto Ros 2011), and is optimal: if $\kappa^{<\kappa} > \kappa$, then $\sqsubseteq_\kappa$ is not invariantly universal. This is joint work with Heike Mildenberger.