

## 1. Problem set for “Models of set theory I”

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**Problem 1.** Let  $R$  be a (set-like) well-founded relation on a class  $C$ . Show that every non-empty subclass of  $C$  has an  $R$ -minimal element. You may take recursion on the natural numbers for granted.

**Problem 2.** Prove from Zorn’s lemma that every set can be well-ordered.

**Problem 3.** Let  $(x, y) := \{\{x\}, \{x, y\}\}$  denote the ordered pair and  $A \times B = \{(x, y) : x \in A, y \in B\}$  the product of classes  $A, B$ . Prove that  $A \times B$  is a set if  $A$  and  $B$  are sets. State which axioms you use.

**Problem 4.** Prove Theorem 2.2 (Transfinite Induction).

Please hand in your solutions 13 April before the lecture