Higher Set Theory - Classical and Ordinal Computability

Exercise Sheet 8
due on Tuesday, 31 May 2011

17. The Gödel pairing function is a bijective map $G : \mathrm{Ord} \times \mathrm{Ord} \to \mathrm{Ord}$ s.t. $G(\alpha, \beta) < G(\gamma, \delta)$ iff one of the following conditions hold:
    - $\max\{\alpha, \beta\} < \max\{\gamma, \delta\}$
    - or $\max\{\alpha, \beta\} = \max\{\gamma, \delta\}$ and $\alpha < \gamma$
    - or $\max\{\alpha, \beta\} = \max\{\gamma, \delta\}$ and $\alpha = \gamma$ and $\beta < \delta$.

    Show that G is OTM-computable.

    (4 points)

18. Show that the halting problem for classical register machines is an OTM-decidable set.

    (4 points)

19. Give an OTM-algorithm that computes addition on ω but fails to compute addition on Ord.

    (4 points)