Higher Set Theory - Classical and Ordinal Computability

Exercise Sheet 4
due on Tuesday, 3 May 2011

9. (a) Show that the remainder function $\mathsf{Rem} : \omega \times \omega \to \omega$,

$$\mathsf{Rem}(x, y) = \text{"the remainder of the division of x by y"}$$

is primitive recursive.

(b) Show that the Gödel β-function $\beta : \omega \times \omega \times \omega \to \omega$,

$$\beta(c, d, i) = \mathsf{Rem}(c, 1 + (i + 1)d)$$

is primitive recursive.

(c*) *(optional, no points awarded)* Show that for an arbitrary sequence $a_0, a_1, \ldots, a_{n-1} \in \omega$ there are $c, d \in \omega$ such that for $i = 0, \ldots, n - 1$ we have

$$\beta(c, d, i) = a_i.$$

Hint: Use the Chinese Remainder Theorem which states that for such $a_0, \ldots, a_{n-1} \in \omega$ and pairwise relatively prime numbers $m_0, \ldots, m_{n-1} \in \omega$ there is a number $c \in \omega$ s.t.

$$c \equiv a_i \mod m_i$$

(4 points)

10. (a) Show that there is a register computable function $F : \omega \times \omega \to \omega$ s.t.

$$F(n, x) = F_n(x)$$

where $F_n : \omega \to \omega$ is the n-th unary primitive recursive function in some feasible enumeration.

(b) Show, by a diagonal argument, that there is a register computable function that is not primitive recursive.

(8 points)