Higher Set Theory - Classical and Ordinal Computability

Exercise Sheet 3
due on Thursday, 28 April 2011

6. Find a computable bijection $F : \omega \times \omega \to \omega$.

(4 points)

7. Universal register machines use a coding of finite tuples of words over $\tilde{\mathbb{A}}^*$ into a single word in $\tilde{\mathbb{A}}^*$. Such a coding has been outlined in the lecture. Show that the i-th projection (i.e. the function that retrieves from a word w that codes $(w_0, w_1, \ldots, w_{n-1})$ the word w_i if $i < n$ and returns $\Box$ else) is computable.

(4 points)

8. Show that the 1-reducibility relation $\leq_1$ induces the structure of an *upper semilattice* on $\mathcal{P}(\mathbb{A}^*)$, i.e. that $(\mathcal{P}(\mathbb{A}^*), \leq_1)$ has the following properties:

   (a) Transitivity

   (b) Reflexivity

   (c) Existence of joins (least upper bounds): For any two sets $A, B \subseteq \mathbb{A}^*$ there is a set C with

   - $A \leq_1 C$ and $B \leq_1 C$
   - For any D with $A \leq_1 D$ and $B \leq_1 D$ we have $C \leq_1 D$.

(4 points)