HIGHER SET THEORY

FORMAL DERIVATIONS AND NATURAL PROOFS EXERCISE SHEET 3

1. Show that models of Peano Arithmetic have definable Skolem functions.

2. Converting a formula from conjunctive normal form to disjunctive normal form or the other way around may increase the size of the formula. Show that this increase is bounded, that is, show that if we convert $\phi = \bigvee_{i<m}(\bigwedge_{j<n_i} L_{ij})$ to $\phi = \bigwedge_{i<m'}(\bigvee_{j<n'_i} L'_{ij})$, then $m' \leq 2^m$.

3. Consider the theory of Peano arithmetic with addition. The axioms can be described as follows

1. $\forall x \forall y (\mathsf{eq}(x, x))$
2. $\forall x \forall y (\mathsf{eq}(x, y) \to \mathsf{eq}(y, x))$
3. $\forall x \forall y \forall z (\mathsf{eq}(x, y) \wedge \mathsf{eq}(y, z) \to \mathsf{eq}(x, z))$
4. $\forall x \forall y (\mathsf{eq}(x, y) \wedge \mathsf{nat}(x) \to \mathsf{nat}(y))$
5. $\mathsf{nat}(0)$
6. $\forall x (\mathsf{nat}(x) \to \mathsf{nat}(\mathsf{succ}(x)))$
7. $\forall x (\neg \mathsf{eq}(\mathsf{succ}(x), 0))$
8. $\forall x \forall y (\mathsf{eq}(\mathsf{succ}(x), \mathsf{succ}(y)) \to \mathsf{eq}(x, y))$
9. $\forall x (\mathsf{eq}(\mathsf{add}(x, 0), x))$
10. $\forall x \forall y (\mathsf{eq}(\mathsf{add}(x, \mathsf{succ}(y)), \mathsf{succ}(\mathsf{add}(x, y))))$

Axioms 1-4 describe the equality relation, 5-8 say what is a natural number the successor function, and 9-10 describe addition. Using resolution calculus prove that addition is commutative.

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