Complexity over arbitrary structures

Komplexitätsbetrachtungen über beliebigen Strukturen

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Complexity over arbitrary structures

1. The uniform model of computation
2. Diagonalization techniques and halting problems
3. Structures with $P \neq NP$
4. Structures and oracles with $P^A \neq NP^A$ and $P^A = NP^A$
5. An idea for the construction of structures with $P = NP$

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The uniform BSS model of computation

Example constants operations relation

A ring: $\mathbb{R} = (\mathbb{R}\,;\, \overbrace{0, 1}^{\text{constants}}\,;\, \overbrace{+, -, \cdot}^{\text{operations}}\,;\, \overbrace{\leq}^{\text{relation}})$

$\mathbb{R} = (\mathbb{R}\,;\, \mathbb{R}\,;\, +, -, \cdot\,;\, \leq)$ (⇨ BSS model)

infinite signature

A computation:

$s := 0;$

for $i := 1$ to n do

(index) integer

any arity

$\{$

$s := s + x_i\,;$

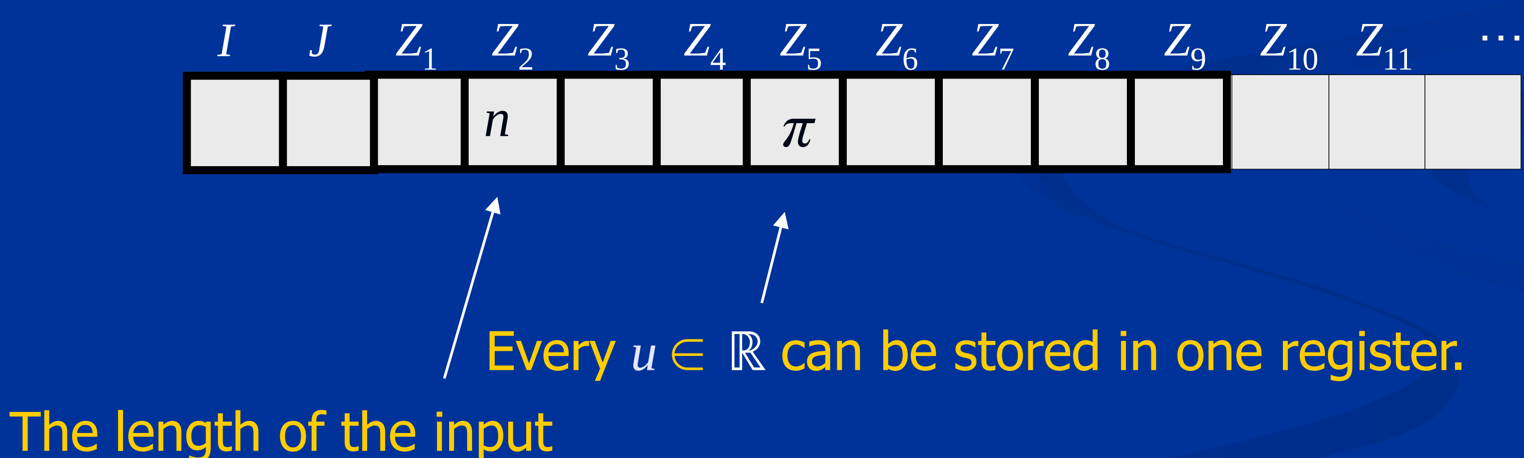
$\}$

real number (element of the structure)

The uniform BSS model of computation

Registers for elements of $\mathbb{R}$: $Z_1, Z_2, Z_3, \ldots$

Registers for indices: I, J

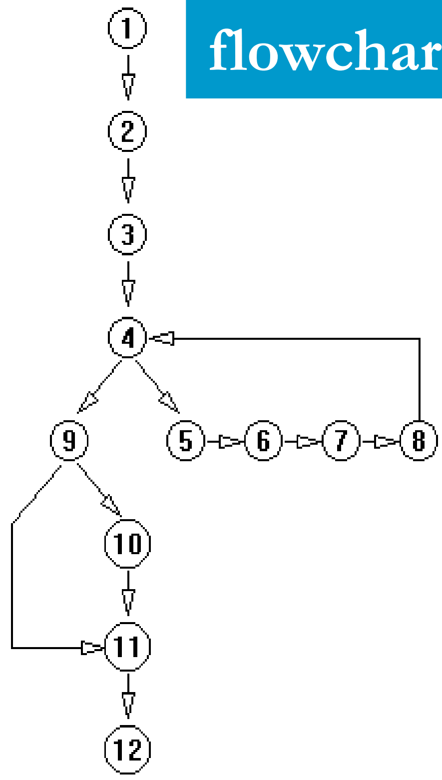


Representations of programs

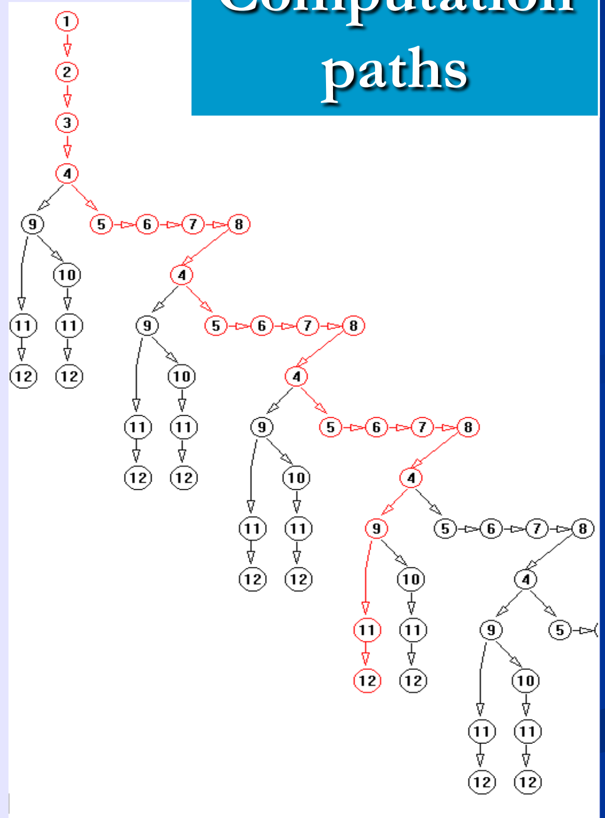
A program

1: Input: $(x_1, \ldots, x_n)$.
Guess: $(y_1, \ldots, y_n) \in \{0, 1\}$.
2: $I_2 := I_1 + 1$;
3: $Z_1 := 1 + Z_1 * Z_{I_2}$;
4: if $I_1 = I_3$ then goto 9
else goto 5;
5: $I_2 := I_2 + 1$;
6: $I_3 := I_3 + 1$;
7: $Z_1 := Z_1 + Z_{I_3} * Z_{I_2}$;
8: goto 4;
9: if $Z_1 = 1$ then goto 11
else goto 10;
10: $Z_1 := 0$;
11: $I_1 := 1$;
12: Output: Z_1.

A flowchart



Computation paths



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The uniform model of computation

A structure: $\Sigma = (U\,;\, c_1,\ldots,c_u\,;\; f_1,\ldots,f_v\,;\; R_1,\ldots,R_w, =)$

$\Sigma = (U\,;\,(c_i)_{i\in F}\,;\,(f_i)_{i\in G}\,;\,(R_i)_{i\in H}\,)$

Computation: $l\!:\; Z_k := f_j(Z_{k_1},\ldots,Z_{k_{m_j}});$

$l\!:\; Z_k := c_j;$

Branching: $l\!:$ if $R_j(Z_{k_1},\ldots,Z_{k_{n_j}})$ then goto l_1 else goto l_2;

$l\!:$ if $Z_k = Z_j$ then goto l_1 else goto l_2;

Copy: $l\!:\; Z_{I_k} := Z_{I_l};$

Index computation: $I_k := 1$; $I_k := I_k + 1$; if $I_k = I_j$ then goto l_1 else goto l_2;

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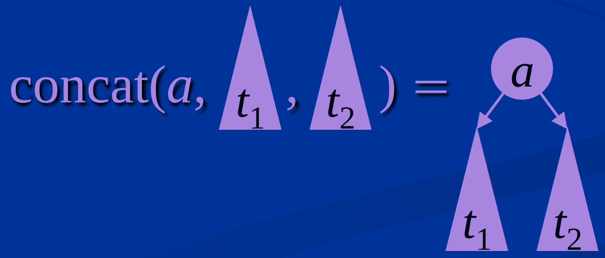
Examples for several structures

$\mathbb{Z}_2 = (\{0, 1\}; 0, 1; +, \cdot\ ; =)$ (⇨ Turing machines)

$\mathbb{R} = (\mathbb{R}; \mathbb{R}; +, -, \cdot\ ; \leq)$ (⇨ BSS model)

$\Sigma_{\text{string}} = (\{0, 1\}^*; \varepsilon, 0, 1; \text{add}, \text{sub}_l, \text{sub}_r; =)$

$\Sigma_{\text{tree}} = (\text{tree}(\mathbb{R}); \text{nil}; \text{concat}, \text{root}, \text{sub}_l, \text{sub}_r; =)$



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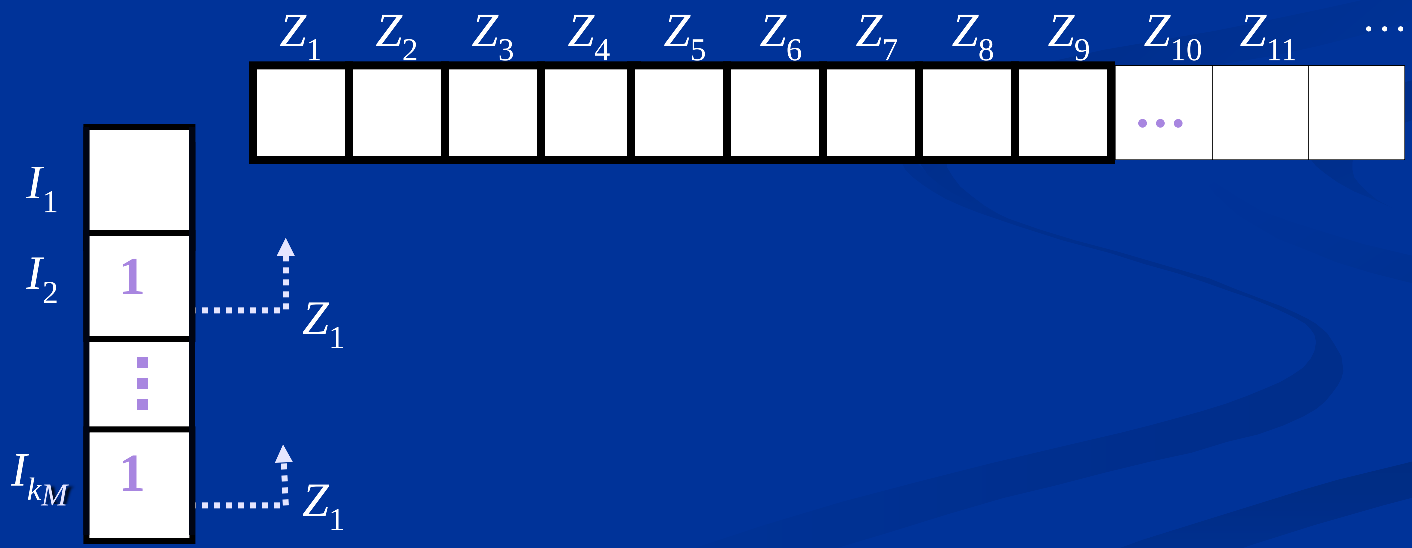
The machine and the input

The input: $(Z_1,\ldots, Z_n) := (x_1,\ldots, x_n);\ I_1 := n;\ \ I_2 := 1;\ldots ;\ I_{k_M} := 1;$



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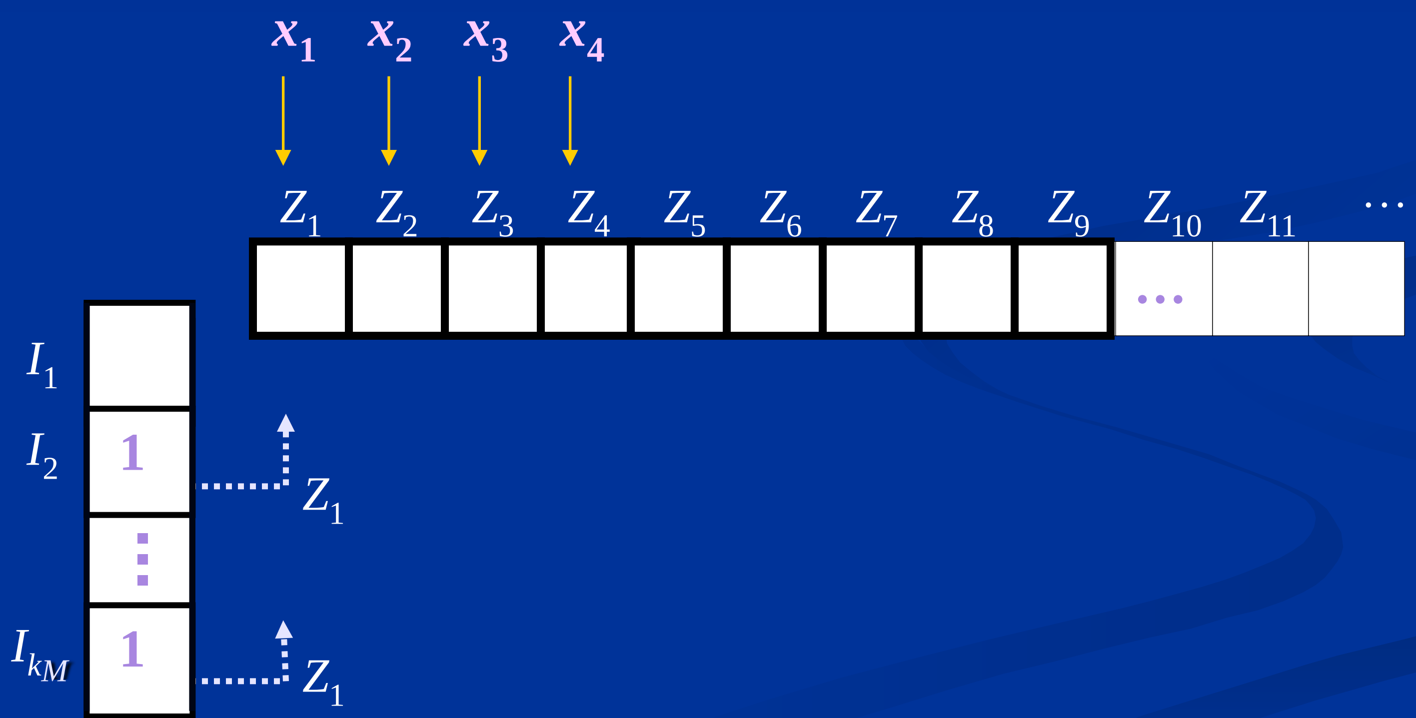

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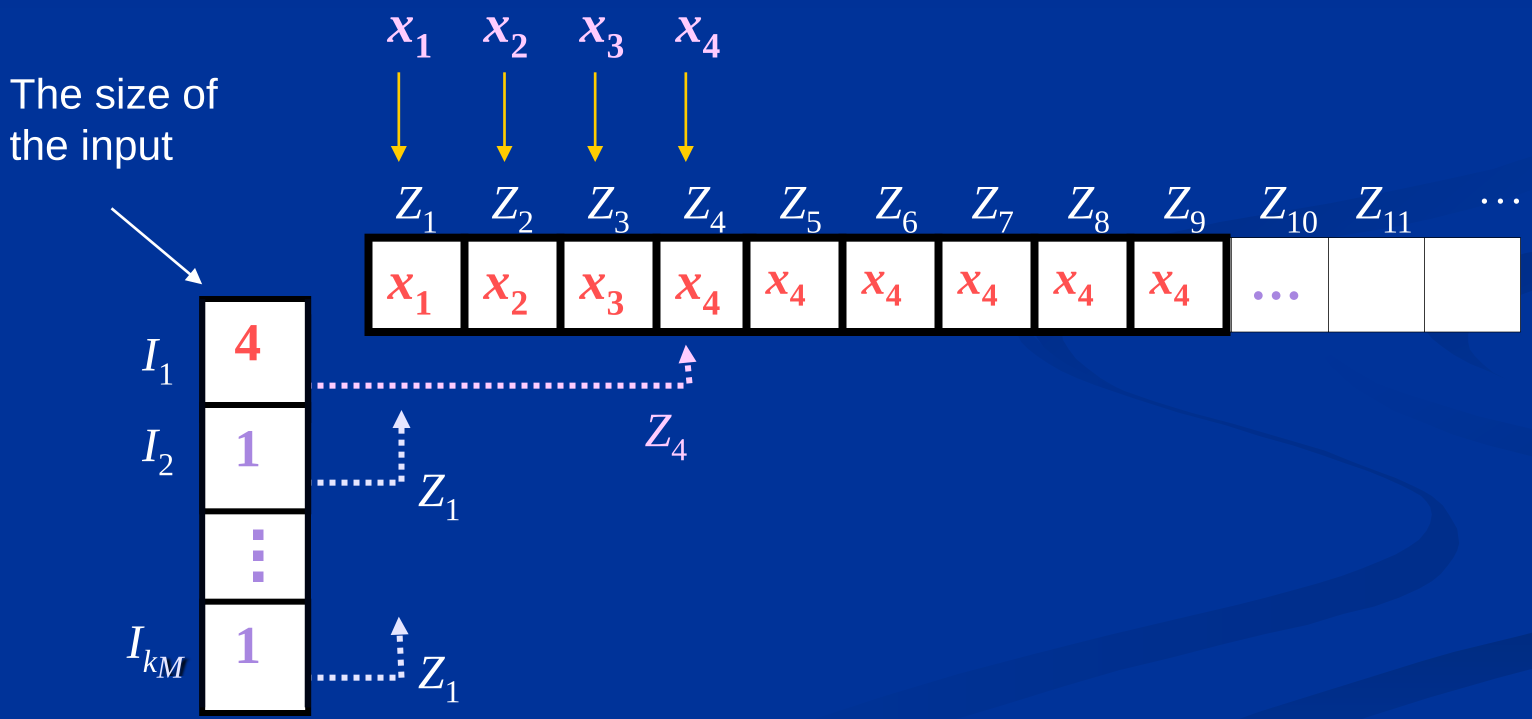
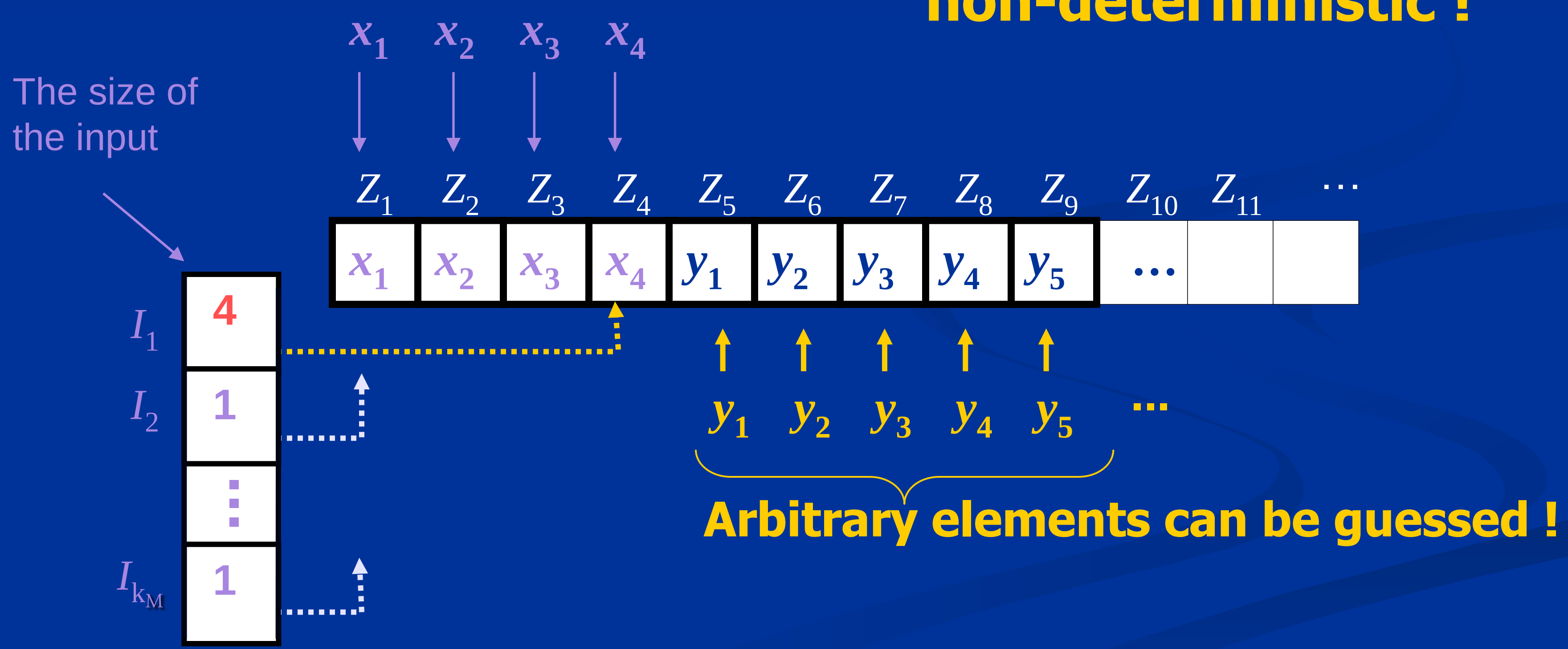


The non-deterministic machines
(input and guessing)

The guessing: $(Z_{n+1},\ldots,Z_{n+m}) := (y_1,\ldots,y_m) \in U^m$

non-deterministic !



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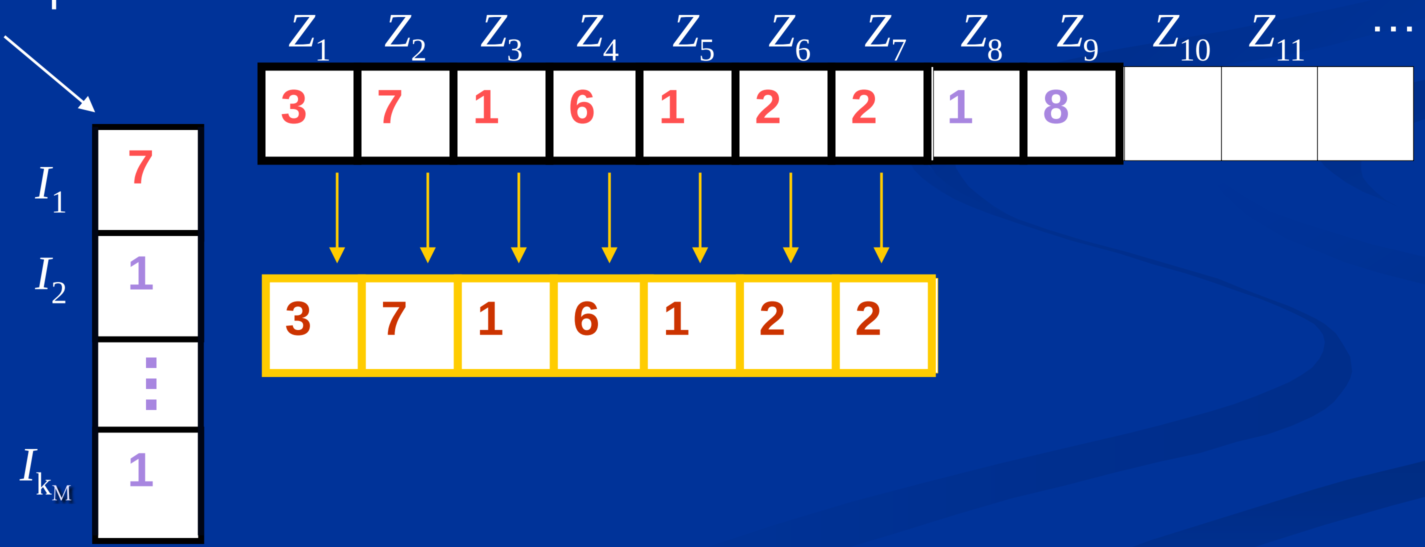
The output

The output: $(Z_1, \ldots, Z_{I_1})$

The size of the output



The class DEC_Σ

Decidable problems

$B \subseteq U^\infty$, $a, b \in U$ are constants with $a \neq b$.

$B \in \mathrm{DEC}_\Sigma$

- if the characteristic function is computable,

if there is a machine with

Input: $(x_1, \ldots, x_n) \in U^\infty$;

Output a (or halt) if $(x_1, \ldots, x_n) \in B$. Acceptance.

Output b (or no halt) if $(x_1, \ldots, x_n) \notin B$. Deterministic rejection.

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Halting problems for Σ

$$H_\Sigma = \{(\underbrace{x_1,\ldots,x_n}_{\boldsymbol{x}}, Code(M)) \mid$$

$$\boldsymbol{x} \in U^\infty \;\;\&\;\; M \text{ is a deterministic } \Sigma\text{-machine}$$

$$\&\;\; M \text{ halts on } \boldsymbol{x}\}$$

$$H_\Sigma^{\text{spec}} = \{Code(M) \mid M \text{ is a deterministic } \Sigma\text{-machine}$$

$$\&\;\; M \text{ halts on } Code(M)\}$$

Diagonalization techniques

The undecidability of the Halting problem H_Σ (for Turing machines)

1. The set of machines is countable. Assume that H_Σ^{spec} is decidable.

Halt?	bin(1)	…	bin(i)	…	bin(j)	…	…
M_1	yes / no						
:		…					
M_i			yes				
:				…			
M_j					no		
:						…	
:							
M								↯

⇒ There is an M recognizing the complement of H_Σ^{spec}. ↯

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:				…			
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:						…	
:							
M							

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Halt?	bin(1)	⋯	bin(i)	⋯	bin(j)	⋯	⋯
M_1	yes / no						
⋮		⋯					
M_i			yes				
⋮				⋯			
M_j					no		
⋮						⋯	
⋮							
M	no / yes	⋯	no	⋯	yes	⋯	⋯

⇒ There is an M recognizing the complement of H_Σ^{spec}. ↯

Diagonalization techniques

The undecidability of the Halting problem H_Σ (for $(\mathbb{R}; \mathbb{R}; +, -, \cdot; \leq)$)

2. The codes of machines are ordered. Assume that H_Σ^{spec} is decidable.

Halt?	…	…	$Code(M_i)$	…	$Code(M_j)$	…	…
:	…						
:		…					
M_i			yes				
:				…			
M_j					no		
:						…	
:							
M	…	…	no	…	yes	…	…

⇒ There is an M recognizing the complement of H_Σ^{spec}. ⚡

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Diagonalization techniques

The undecidability of the Halting problem H_Σ (for any structure)

3. Σ arbitrary (We can generalize the result; CCA 2008.)

Assume: H_Σ is decidable.

$\Rightarrow$ H_Σ^{spec} is decidable.

$\Rightarrow$ The complement of H_Σ^{spec} is semi-decidable by a Σ-machine M.

$\Rightarrow$ M halts on $Code(M)$

$\Leftrightarrow$ M does not halt on $Code(M)$.

$\Rightarrow$ ↯

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The class P_Σ

Computation in polynomial time

A machine M decides a problem in polynomial time

if there is some polynomial p_M such that

$$M \text{ halts for } \boldsymbol{x} = (x_1, \ldots, x_n) \text{ within } p_M(n) \text{ steps.}$$

Each instruction is executed within one fixed time unit.

⇨ $P_\Sigma \subseteq DEC_\Sigma$ ($P_\Sigma \triangleq$ problems are decidable in polynomial time)

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The class NP_Σ

The non-deterministic instructions

The non-deterministic:

guess(Z_k); Arbitrary elements can be guessed!

⇨ $P_\Sigma \subseteq NP_\Sigma$

Non-deterministic acceptance: output a by means of guessed elements.

Non-deterministic rejection: if the input cannot be accepted.

$NP_\Sigma \not\subseteq DEC_\Sigma \Rightarrow P_\Sigma \neq NP_\Sigma$ wegen $P_\Sigma \subseteq DEC_\Sigma$ und $P_\Sigma \subseteq NP_\Sigma$.

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Some $P_\Sigma \stackrel{?}{=} NP_\Sigma$ problems for several structures

Σ	$P_\Sigma = NP_\Sigma$?
$(\mathbb{C};\mathbb{C};+,-,\cdot;\ =)$	**?**
$(\mathbb{R};\mathbb{R};+,-,\cdot;\ \leq)$	**?**
$(\mathbb{R};\mathbb{R};+,-,\cdot;\ =)$	no ($\leq$)
$(\mathbb{Q};\mathbb{Q};+,-,\cdot;\leq)$, $(\mathbb{Q};\mathbb{Q};+,-,\cdot;=)$	no (rational square numbers)
$(\mathbb{R};\mathbb{R};+,-;\ \leq)$	**?**
$(\mathbb{R};\mathbb{R};+,-;\ =)$	no (Meer / Koiran)
$(\mathbb{Z};\mathbb{Z};+,-;\leq)$, $(\mathbb{Z};\mathbb{Z};+,-;=)$	no (even integers)
$(\mathbb{Z};1;(\varphi_s)_{s\in\mathbb{Z}};=)$ $\quad \varphi_s(x)=sx$	no (no NP-complete problem)

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Example for $P_\Sigma \neq NP_\Sigma$

$\Sigma = \mathbb{Z} = (\mathbb{Z}; 0, 1; \cdot, +, -; =)$

$A = \{(x,\dots,x) \mid \exists y\ (x = y^2)\}.$

$A \in NP_\Sigma$ (we can guess $y \in \mathbb{Z}$).

$A \notin P_\Sigma$:

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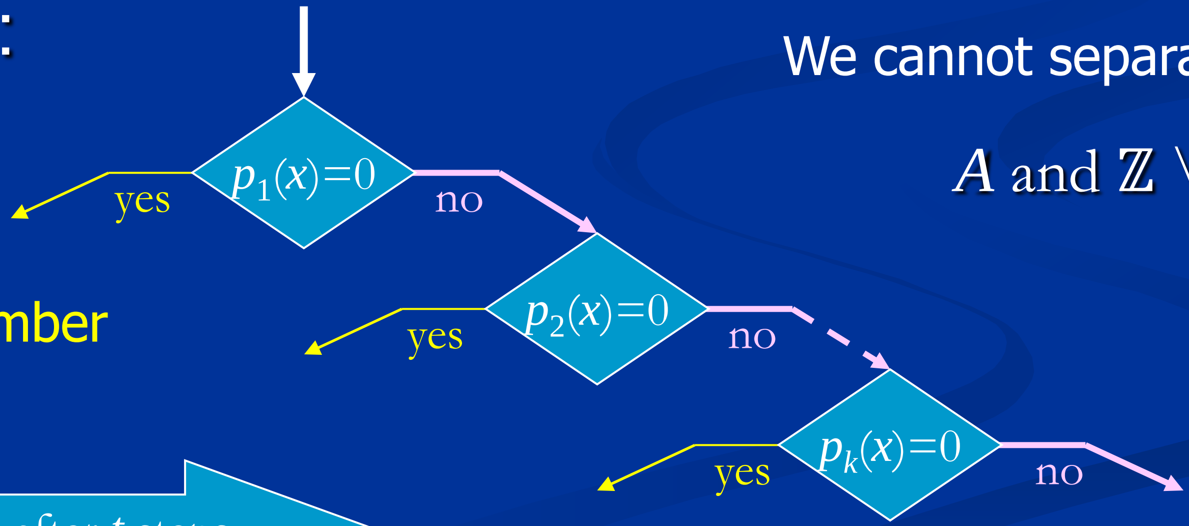

We cannot separate

A and $\mathbb{Z} \setminus A$.

Only a finite number of zeros.

The machine halts after t steps.

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Some P_Σ-NP_Σ problems

for structures over numbers

Σ	$P_\Sigma = DNP_\Sigma$?	$DNP_\Sigma = NP_\Sigma$?
$(\mathbb{C}\,;\,\mathbb{C}\,;\,+,-,\cdot\,;\ =)$	**?**	**?**
$(\mathbb{R}\,;\,\mathbb{R}\,;\,+,-,\cdot\,;\ \leq)$	**?**	**?**
$(\mathbb{R}\,;\,\mathbb{R}\,;\,+,-,\cdot\,;\ =)$	**?**	no ($\leq$)
$(\mathbb{R}\,;\,\mathbb{R}\,;\,+,-\,;\ \leq)$	**?**	yes (Koiran)
$(\mathbb{R}\,;\,\mathbb{R}\,;\,+,-\,;\ =)$	no (Meer)	yes (Koiran)
$(\mathbb{Z}\,;\,\mathbb{Z}\,;\,+,-\,;\,\leq)$	**?**	no (even integers)
$(\mathbb{Z}\,;\,\mathbb{Z}\,;\,+,-\,;\,=)$	no (for groups)	no (even integers)

DN $\triangleq$ digitally non-deterministic:
$y_1,\ldots,y_m \in \{0\,,\,1\}$

Example for $P_\Sigma \neq DNP_\Sigma$

$\Sigma = \mathbb{Z}_{add} = (\mathbb{Z}; 0, 1; +, - ; =)$

D ≙ digitally.

$A = \cup_{n\geq 1}\{(x,\ldots,x) \in \mathbb{Z}^n \mid x < 2^n\}.$

$A \in DNP_\Sigma$ (we can guess the binary code of x: $(y_1,\ldots,y_n) \in \{0,1\}^n$).

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Example for $P_\Sigma \neq DNP_\Sigma$

$\Sigma = \mathbb{Z}_{add} = (\mathbb{Z}; 0, 1; +, - \ ; =)$

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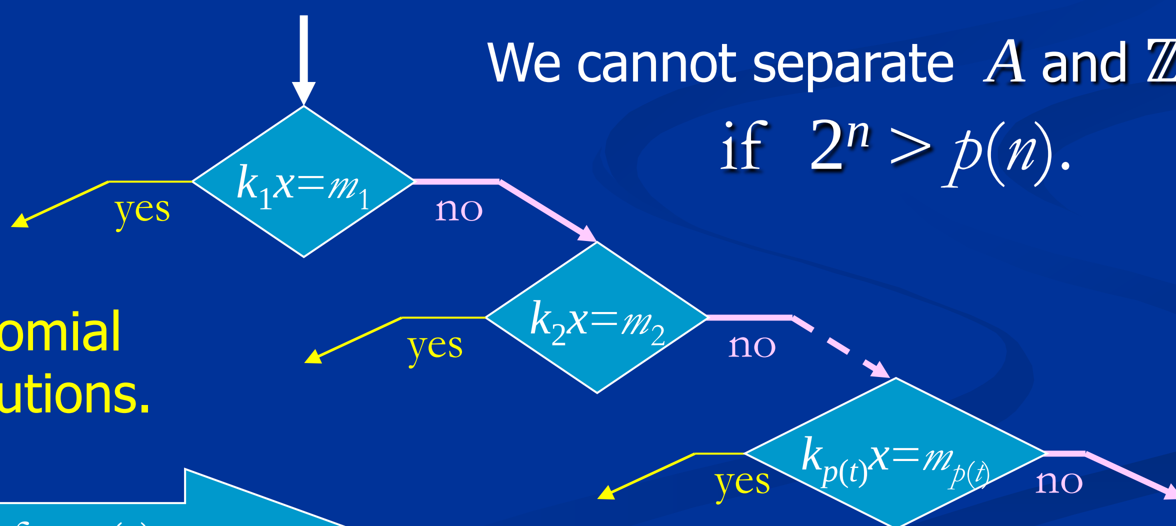
$A \in DNP_\Sigma$ (we can guess the binary code of x: $(y_1, \ldots, y_n) \in \{0,1\}^n$).

$A \notin P_\Sigma$:

We cannot separate A and $\mathbb{Z}^n \setminus A$ if $2^n > p(n)$.



Only a polynomial number of solutions.

The machine halts after $p(n)$ steps

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DEC_Σ , P_Σ , NP_Σ

The size of an input $(x_1,\dots, x_n)$: $\boldsymbol{n}$.

⇨ Every $u \in U$ can be stored in one register.

The execution of any instruction: **one time unit** (one step).

⇨ The execution of one operation $\triangleq$ one time unit.

Computation in polynomial time: **output after $p(n)$ steps** for any input $(x_1,\dots, x_n)$ and some polynomial p.

⇨ $\mathrm{P}_\Sigma \subseteq \mathrm{NP}_\Sigma$

⇨ $\mathrm{P}_\Sigma \subseteq \mathrm{DEC}_\Sigma$

⇨ $\mathrm{NP}_\Sigma \not\subseteq \mathrm{DEC}_\Sigma \;\Rightarrow\; \mathrm{P}_\Sigma \neq \mathrm{NP}_\Sigma$

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NP-complete problems

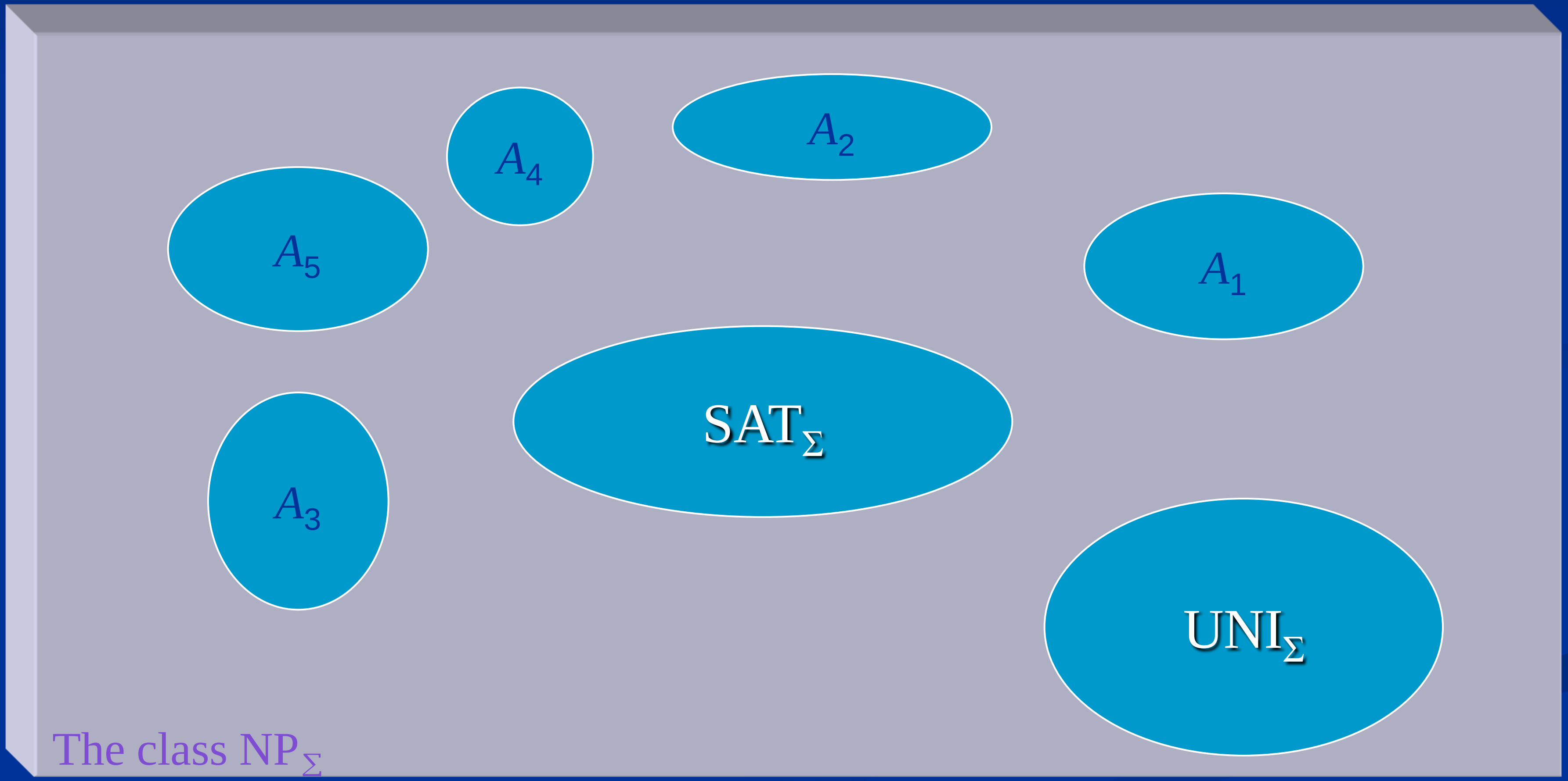
The Satisfiability Problem

$$\mathbf{SAT}_{\Sigma} = \{(\boldsymbol{x}, code(\psi)) \in U^{\infty} \mid \psi \text{ quantifier-free } (\neg, \vee, \wedge)\text{-formulae } \& \ \Sigma \models \exists \boldsymbol{Y}\, \psi(\boldsymbol{x}, \boldsymbol{Y})\}$$

The NP-complete problem recognized by a usual universal machine

$$\mathbf{UNI}_{\Sigma} = \{(b,\ldots, b, x_1,\ldots, x_n, Code(M)) \in U^{t+n+k} \mid M \text{ is a non-deterministic } \Sigma\text{-machine } \& \ M \text{ accepts } (x_1,\ldots, x_n) \text{ within } t \text{ steps}\} \subseteq U^{\infty}$$

The meaning of these NP-complete problems



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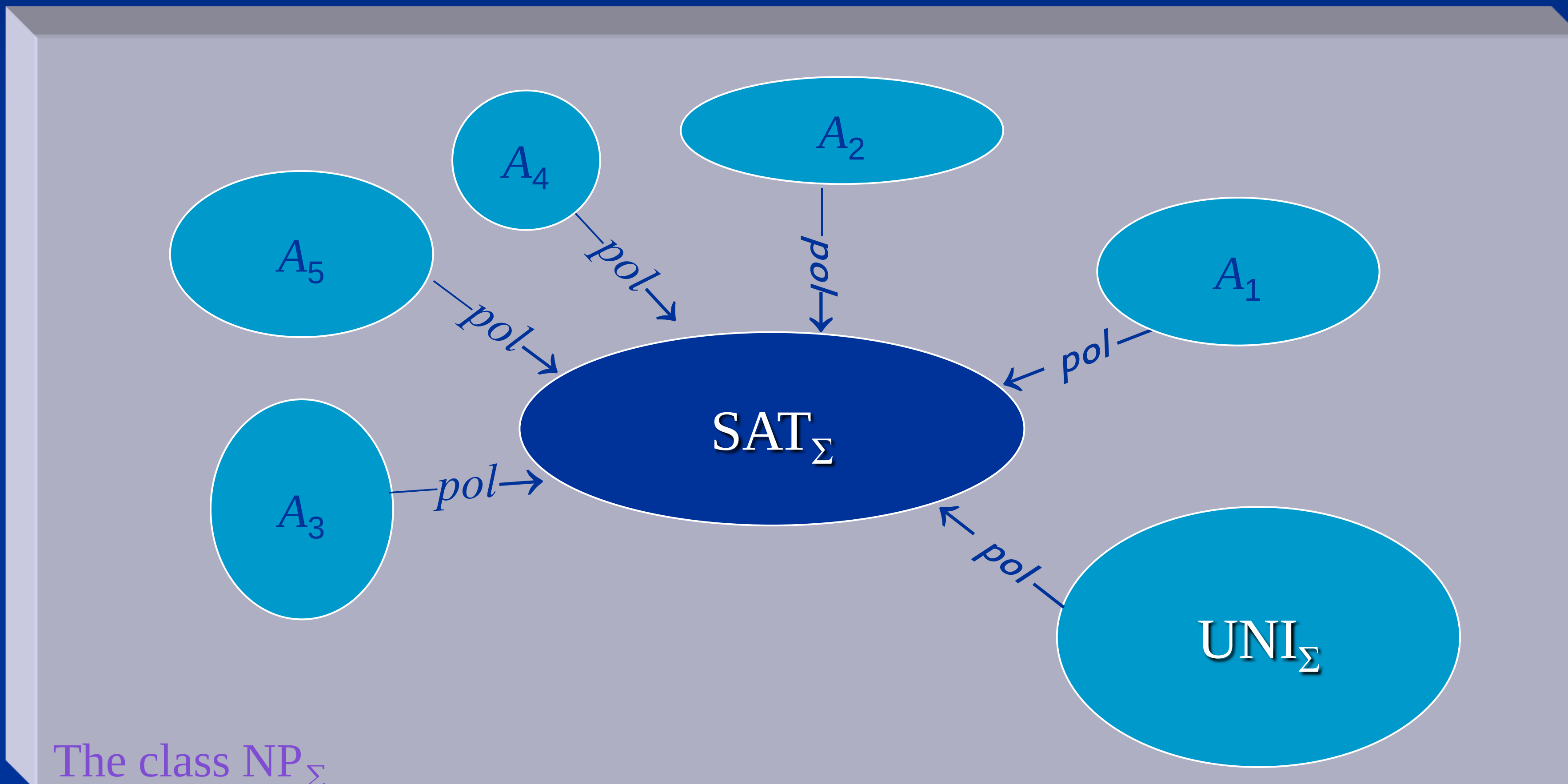
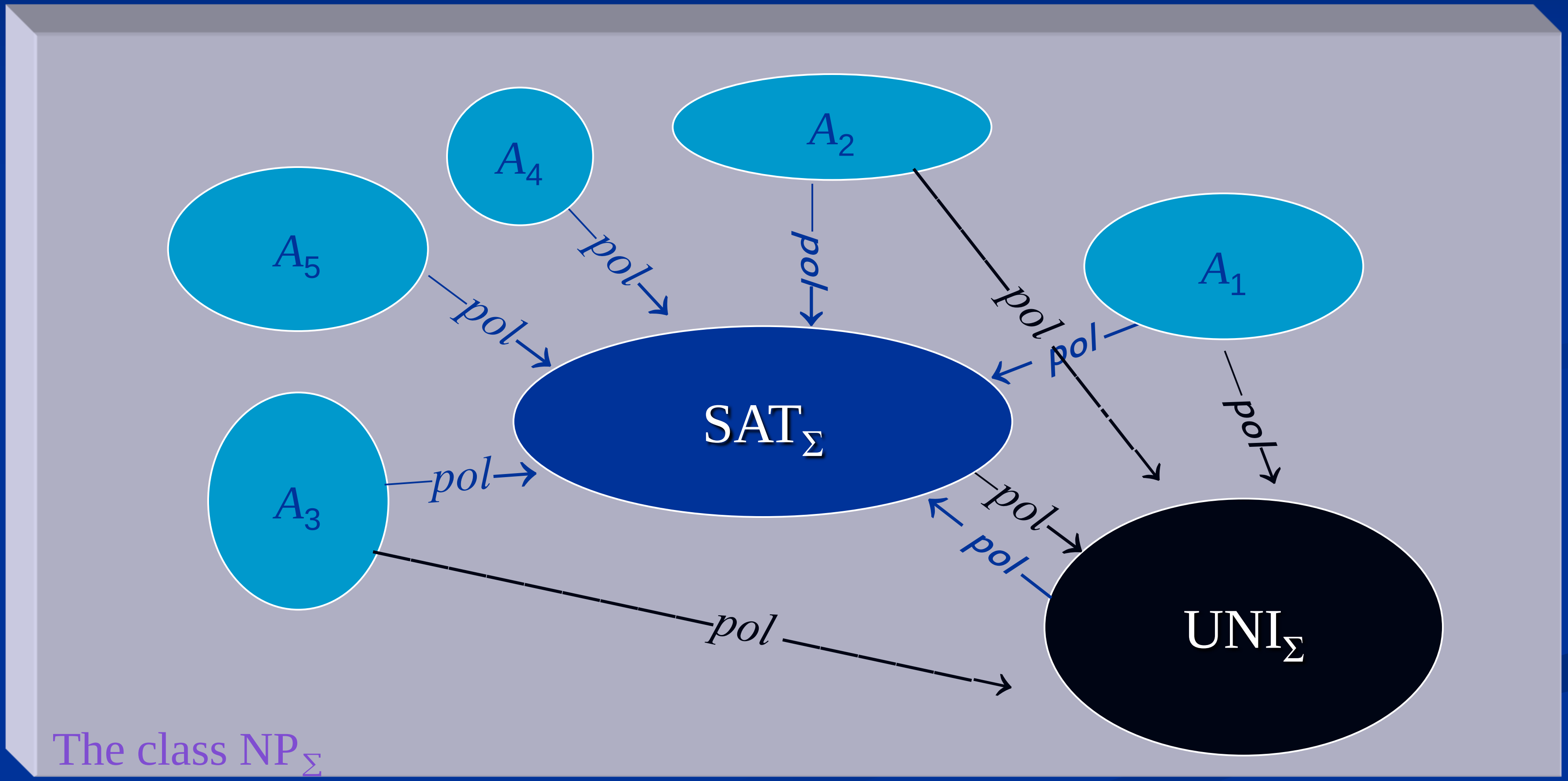
The meaning of these NP-complete problems

Any problem can be reduced to SAT_{Σ} and to UNI_{Σ} in polynomial time.



The meaning of these NP-complete problems

Any problem can be reduced to SAT_Σ and to UNI_Σ in polynomial time.



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Properties of SAT_Σ and UNI_Σ

$SAT_\Sigma \in NP_\Sigma$. SAT_Σ is NP_Σ-complete.

$SAT_\Sigma \in P_\Sigma \quad \Rightarrow P_\Sigma = NP_\Sigma$

$SAT_\Sigma \notin DEC_\Sigma \quad \Rightarrow P_\Sigma \neq NP_\Sigma$

$UNI_\Sigma \in NP_\Sigma$. UNI_Σ is NP_Σ-complete.

$UNI_\Sigma \in P_\Sigma \quad \Rightarrow P_\Sigma = NP_\Sigma$

$UNI_\Sigma \notin DEC_\Sigma \quad \Rightarrow P_\Sigma \neq NP_\Sigma$

Oracle machines

Oracle query:

$$l\text{: if } \underbrace{(Z_1,\ldots,Z_{I_1})} \in B \text{ then goto } l_1 \text{ else goto } l_2\,;$$

The length can be computed by $I_1 := 1$; $I_1 := I_1 + 1$; ….

B oracle, $B \subseteq U^\infty = \cup_{n\geq 1} U^n$

We will define oracles such that

$$\mathrm{P}_\Sigma{}^{Q} \neq \mathrm{NP}_\Sigma{}^{Q},$$

$$\mathrm{P}_\Sigma{}^{O} = \mathrm{NP}_\Sigma{}^{O}.$$

An oracle Q with $P_\Sigma^Q \neq NP_\Sigma^Q$

Using the undecidability of the Halting problem H_Σ

U infinite,
a finite number of operations and relations,

$\{\alpha_1, \alpha_2, \alpha_3, \ldots\} \subseteq U$ enumerable and decidable.

$Q = Q_\Sigma = \{ (\alpha_t, \boldsymbol{x}, Code(M)) \mid$

$\boldsymbol{x} \in U^\infty$ & M is a deterministic Σ-machine

& $M(\boldsymbol{x})\downarrow^t\}$

M accepts $\boldsymbol{x} = (x_1, \ldots, x_n) \in U^\infty$ within t steps.

Proposition (CCA 2008): $H_\Sigma \in NP_\Sigma^Q \setminus P_\Sigma^Q$. $(P_\Sigma^Q \subseteq DEC_\Sigma)$

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An oracle $\mathcal{O}_\Sigma$ with $P_\Sigma^{\mathcal{O}_\Sigma} = NP_\Sigma^{\mathcal{O}_\Sigma}$

A universal oracle:

$$\mathcal{O} = \mathcal{O}_\Sigma = \{ (\overbrace{b,\ldots,b}^{\in U^t}, \boldsymbol{x}, Code(M)) \mid$$

$$\boldsymbol{x} \in U^\infty \;\; \& \;\; M \text{ is a non-deterministic } \Sigma\text{-machine using } \mathcal{O}$$

$$\& \;\; M(\boldsymbol{x})\downarrow^t \}$$

(cp. also Baker, Gill, and Solovay; Emerson; ... for Turing machines...)

Proposition (CCA 2008): $P_\Sigma^{\mathcal{O}} = NP_\Sigma^{\mathcal{O}}$.

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An oracle $\mathcal{O}_\Sigma$ containing only tuples of length 1 with $P_\Sigma^{\mathcal{O}_\Sigma} = NP_\Sigma^{\mathcal{O}_\Sigma}$?

Structures over strings

$\Sigma = (U^*; \varepsilon, a, b, c_3,\ldots, c_u; \; add, sub_\mathrm{l}, sub_\mathrm{r}, \; f_1,\ldots, f_v; R_1,\ldots,R_w, \; =)$

$(d_1,\ldots, d_k) \in U^k \subset U^\infty$ stored in k registers

!

$s = d_1 \cdots d_k \in U^*$ stored in one register

$d \in U$

$add(s, d) = sd$ $\quad sub_\mathrm{l}(sd) = s$ $\quad sub_\mathrm{r}(sd) = d$

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An oracle $\mathcal{O}_\Sigma$ containing only tuples of length 1 with $\mathrm{P}_\Sigma^{\mathcal{O}_\Sigma} = \mathrm{NP}_\Sigma^{\mathcal{O}_\Sigma}$?

Recall: $\mathrm{P}_\Sigma^{\mathcal{O}_\Sigma} = \mathrm{NP}_\Sigma^{\mathcal{O}_\Sigma}$ and $\mathrm{P}_\Sigma^{\mathcal{Q}_\Sigma} \neq \mathrm{NP}_\Sigma^{\mathcal{Q}_\Sigma}$ for

$$\mathcal{O}_\Sigma = \{\, (\underbrace{b,\ldots,b}_{t\,\times}, \boldsymbol{x}, \mathit{Code}(M)) \mid \boldsymbol{x} \in (U^*)^\infty \;\&\; M \text{ is a non-deterministic } \Sigma\text{-machine using } \mathcal{O}_\Sigma \;\&\; M(\boldsymbol{x})\downarrow^t \}$$

$$\mathcal{Q}_\Sigma = \{\, (\underbrace{b\cdots b}_{t\,\times}, \boldsymbol{x}, \mathit{Code}(M)) \mid \boldsymbol{x} \in (U^*)^\infty \;\&\; M \text{ is a deterministic } \Sigma\text{-machine} \;\&\; M(\boldsymbol{x})\downarrow^t \}$$

Theorem (CCA 2008): There is not an oracle $\mathcal{O}$ with

$$b\cdots b\cdot \mathrm{string}(\boldsymbol{x})\cdot \mathrm{string}(\mathit{Code}(M)) \in \mathcal{O}$$

$$\Leftrightarrow \quad \boldsymbol{x} \in (U^*)^\infty \;\&\; M \text{ is a non-deterministic } \Sigma\text{-machine using } \mathcal{O} \;\&\; M(\boldsymbol{x})\downarrow^t .$$

No set !

Structures with P = NP

An additional relation R
on padded codes
of the elements of a universal oracle O
with $P_{\Sigma}^{O} = NP_{\Sigma}^{O}$

P = NP for

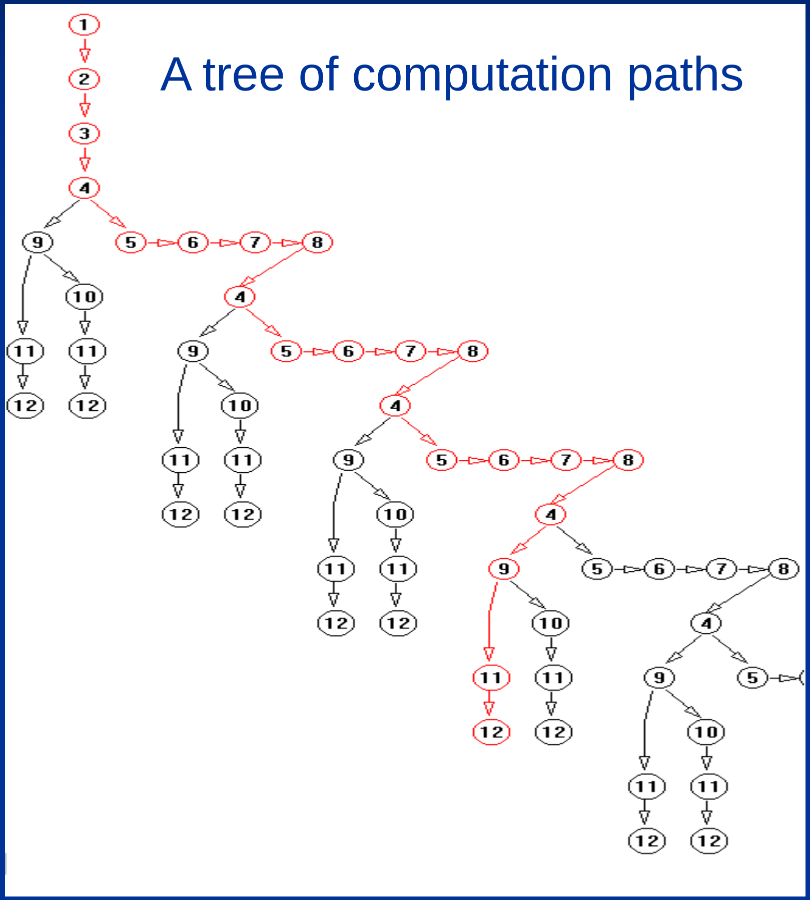
P = NP for

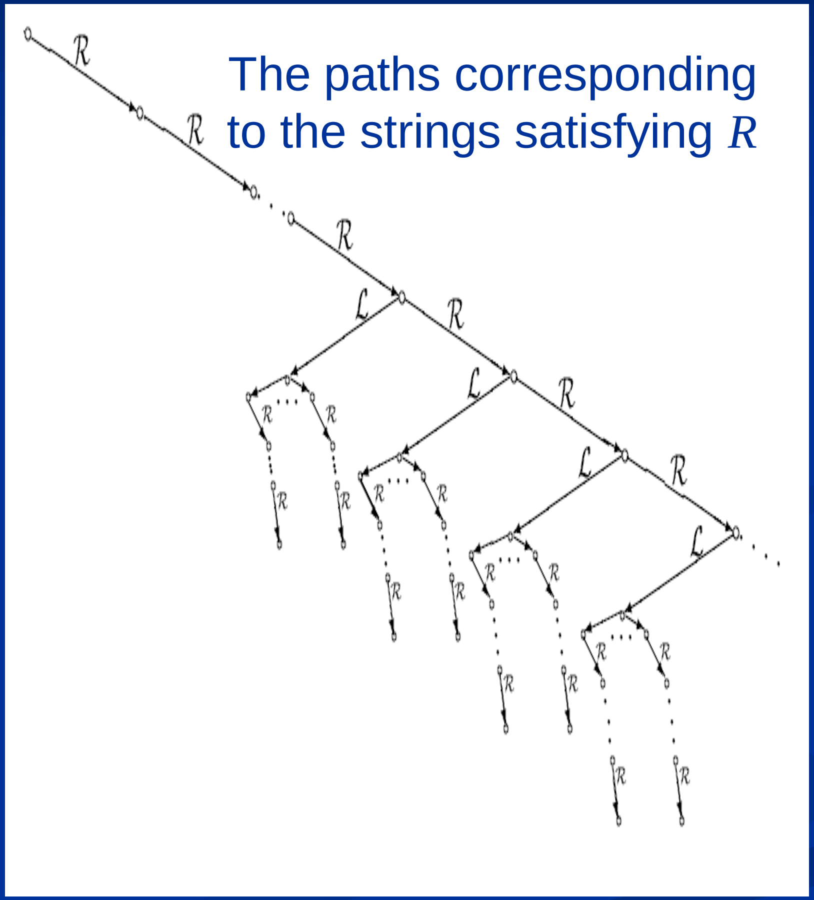
Binary trees
with decidable identity relation
(Gaßner, Dagstuhl 2004)

Strings
with identity relation and relation R recursively defined by means of $SAT_{\Sigma R}$
(Gaßner, CiE 2006)

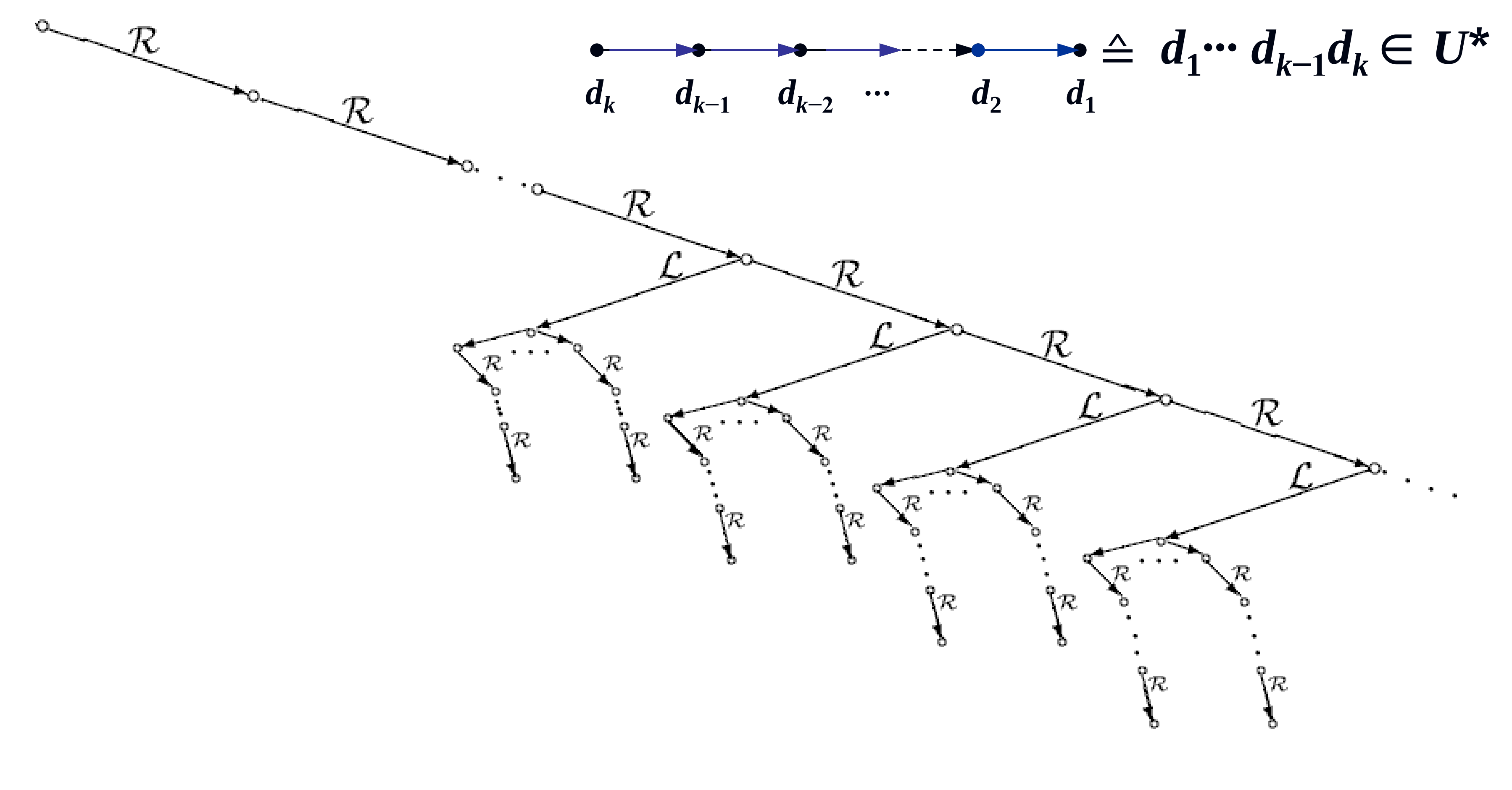
Strings
with operations for adding and deleting the last character
(Gaßner, CiE 2007)

The idea - similar trees





The description of UNI_{Σ_R} by a tree



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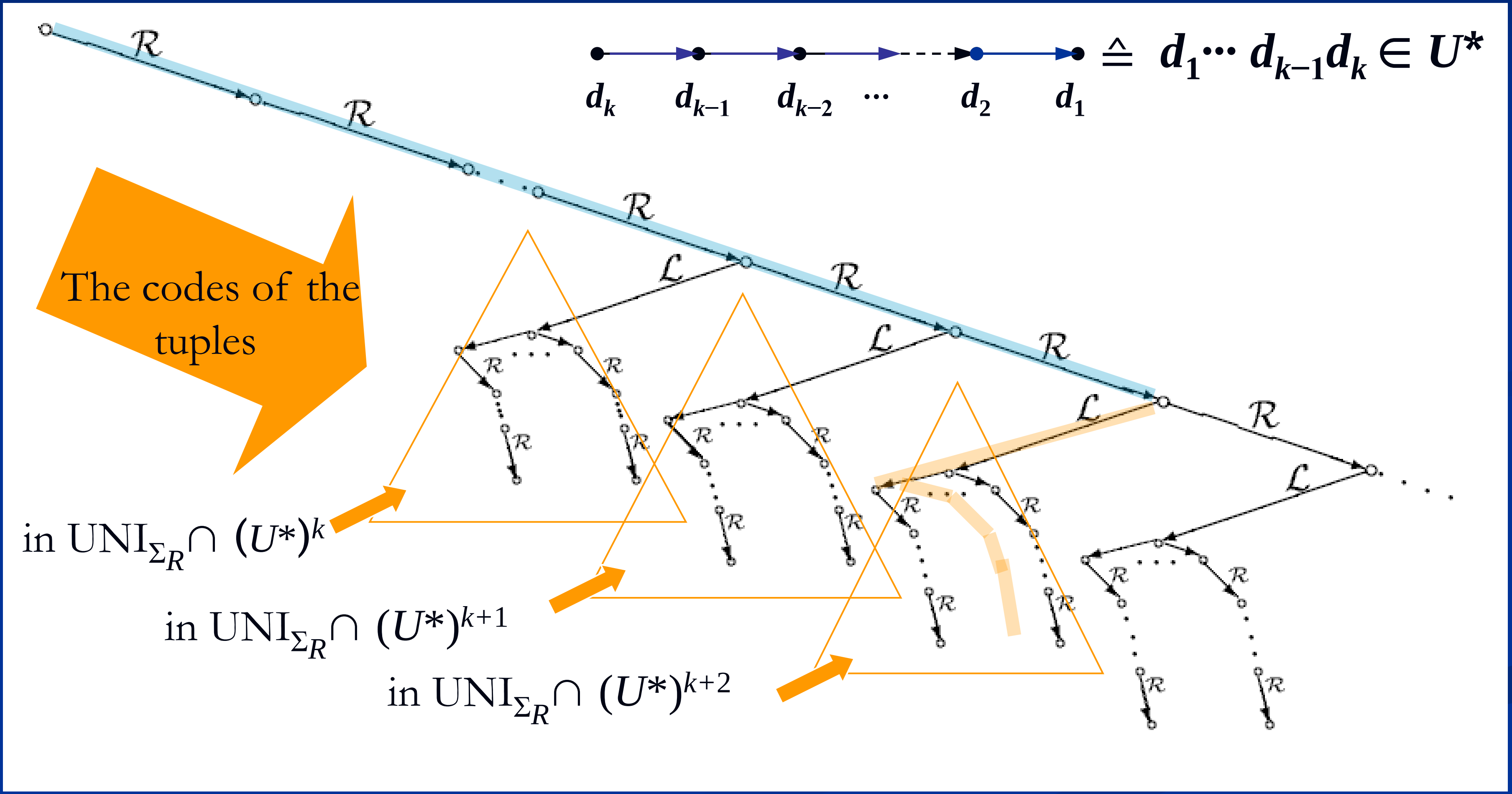

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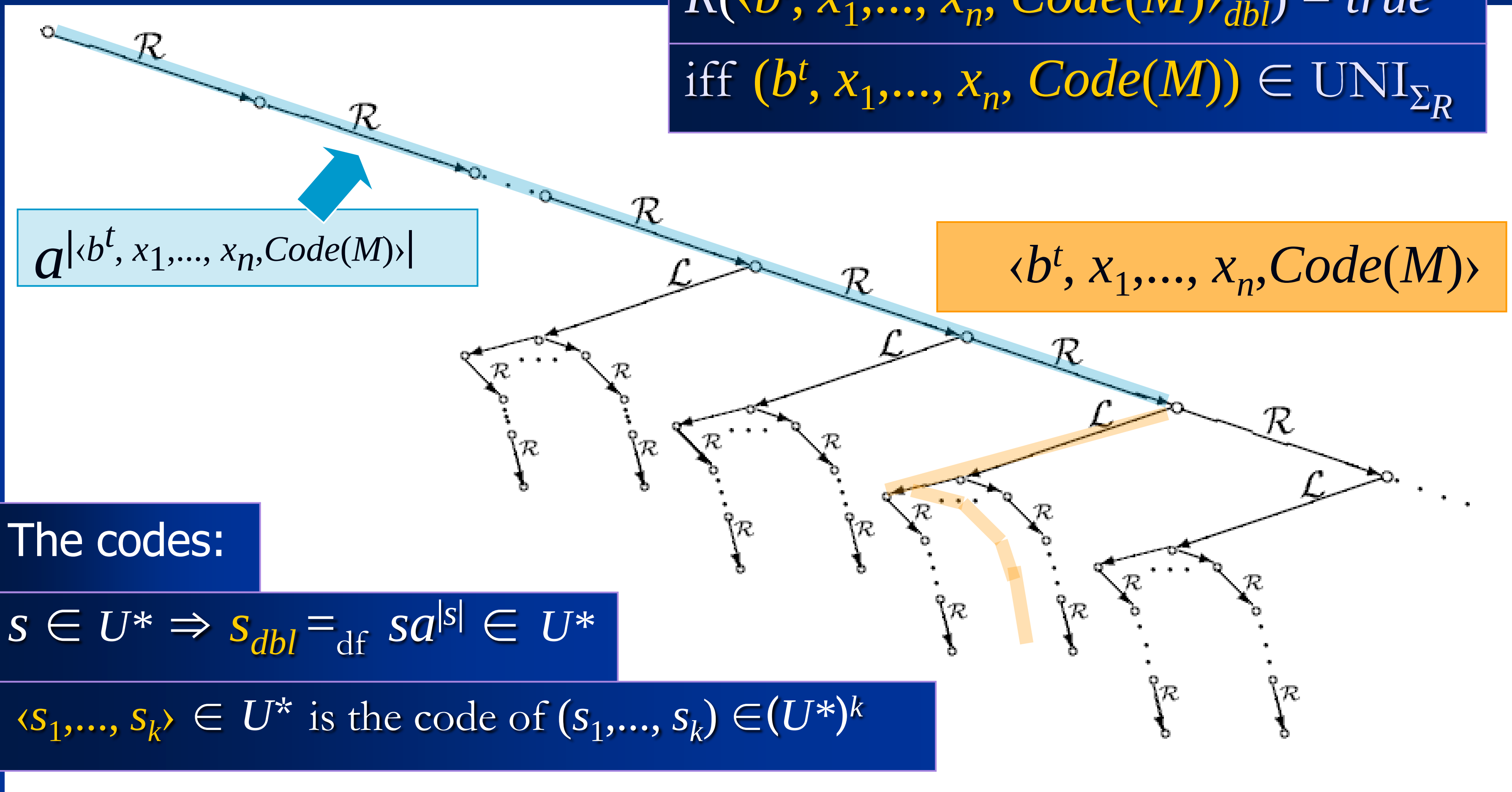
Some R with $\mathrm{UNI}_{\Sigma_R} \in \mathrm{DEC}_{\Sigma_R}$

$R(\langle b^t, x_1,\ldots, x_n, Code(M)\rangle_{dbl}) = true$

iff $(b^t, x_1,\ldots, x_n, Code(M)) \in \mathrm{UNI}_{\Sigma_R}$



The codes:

$s \in U^* \Rightarrow s_{dbl} =_{\mathrm{df}} sa^{|s|} \in U^*$

$\langle s_1,\ldots, s_k\rangle \in U^*$ is the code of $(s_1,\ldots, s_k) \in (U^*)^k$

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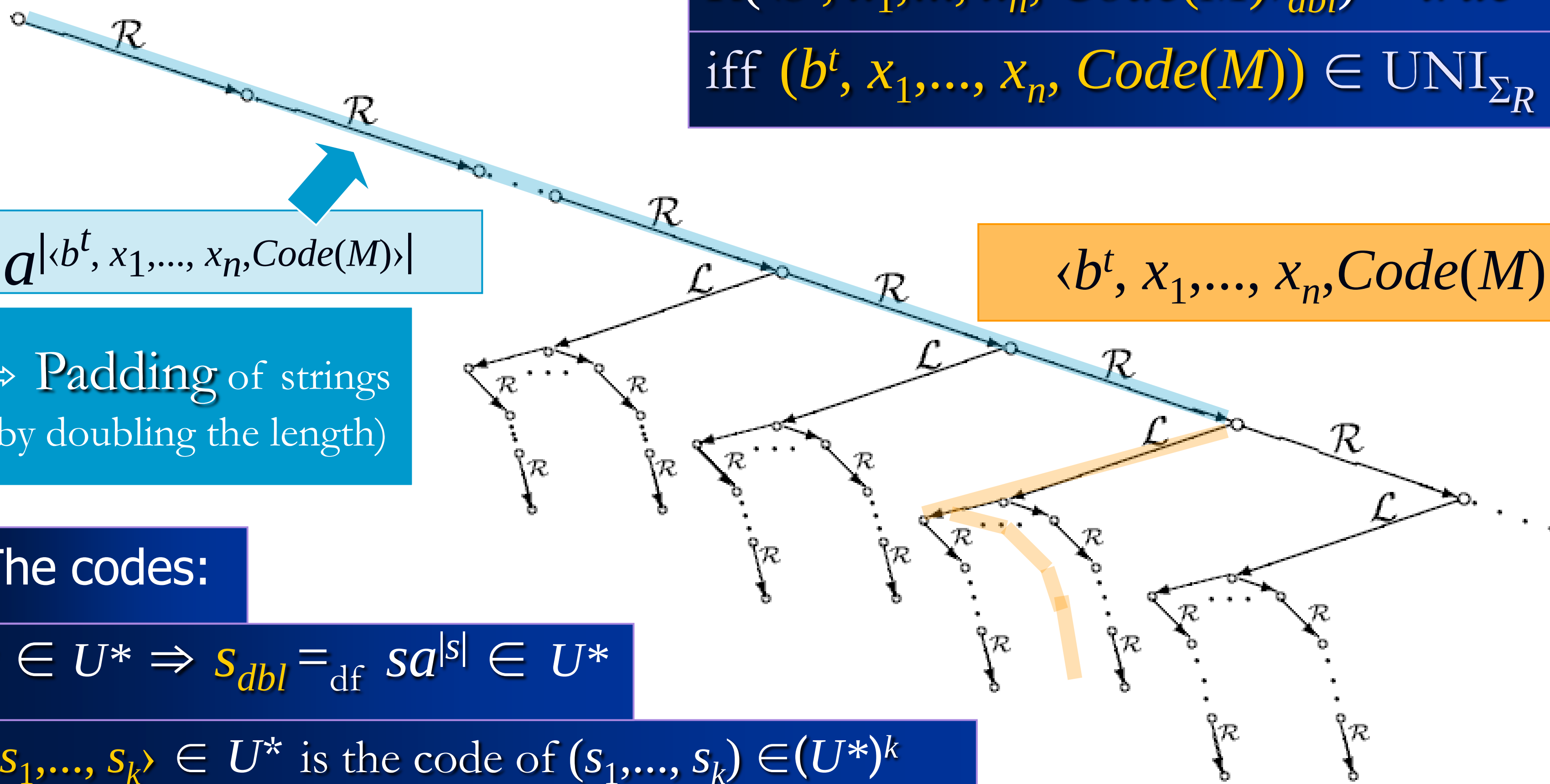

$a^{|\langle b^t, x_1,\ldots, x_n, Code(M)\rangle|}$

$\langle b^t, x_1,\ldots, x_n, Code(M)\rangle$

⇨ Padding of strings (by doubling the length)

The codes:

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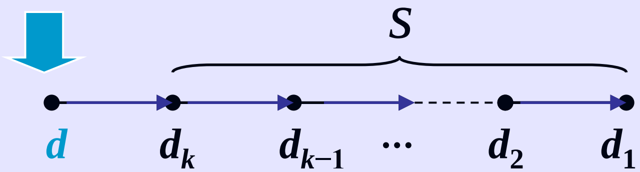
The new operations for slow (!!!) computation

$\Sigma \quad = (U ; c_1,\ldots, c_u; \; f_1,\ldots, f_v; R_1,\ldots, R_w, =),$

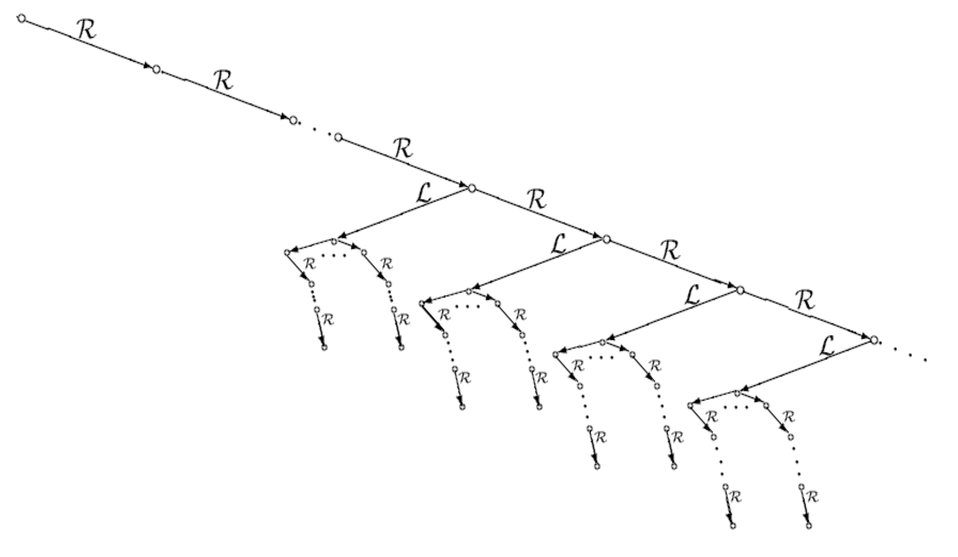
$\Sigma_R = (U^* ; \; \varepsilon, a, b, c_1 ,\ldots, c_u ; \; add, sub_l , sub_r , \; f_1',\ldots, f_v'; R_1',\ldots, R_w', R , =)$

$add(s, d) = sd \qquad sub_l(sd) = s \qquad sub_r(sd) = d \qquad s \in U^*, d \in U$



$s = d_1 \cdots d_{k-1} d_k$

$add(s, d) = d_1 \cdots d_k d$



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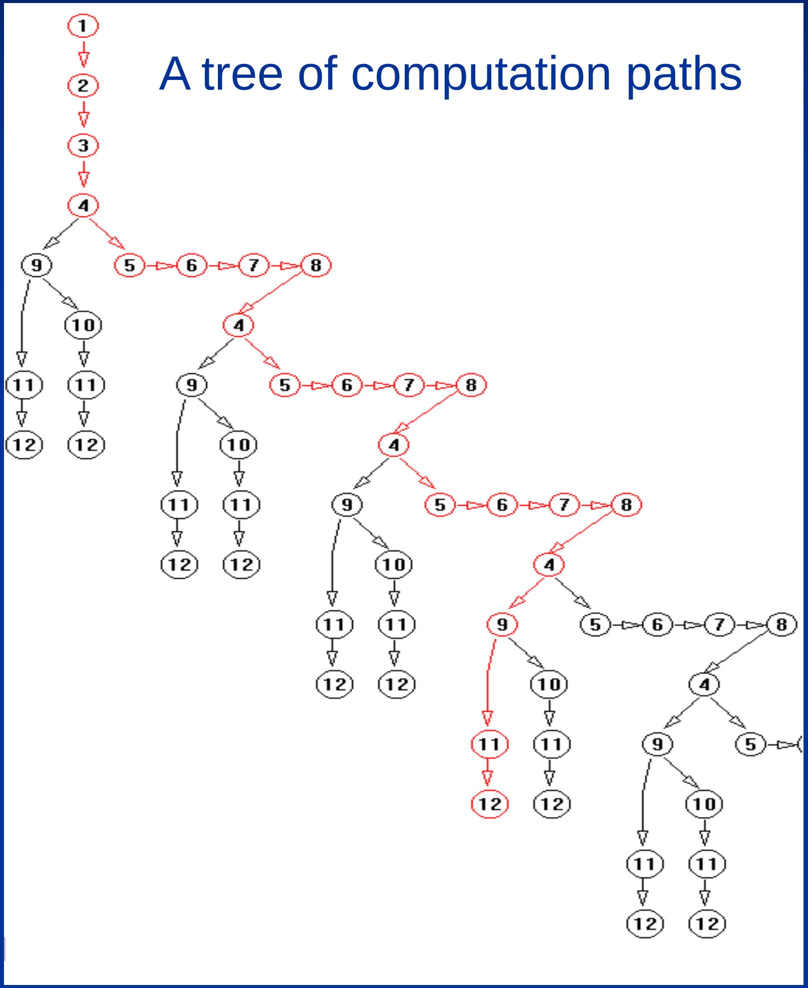
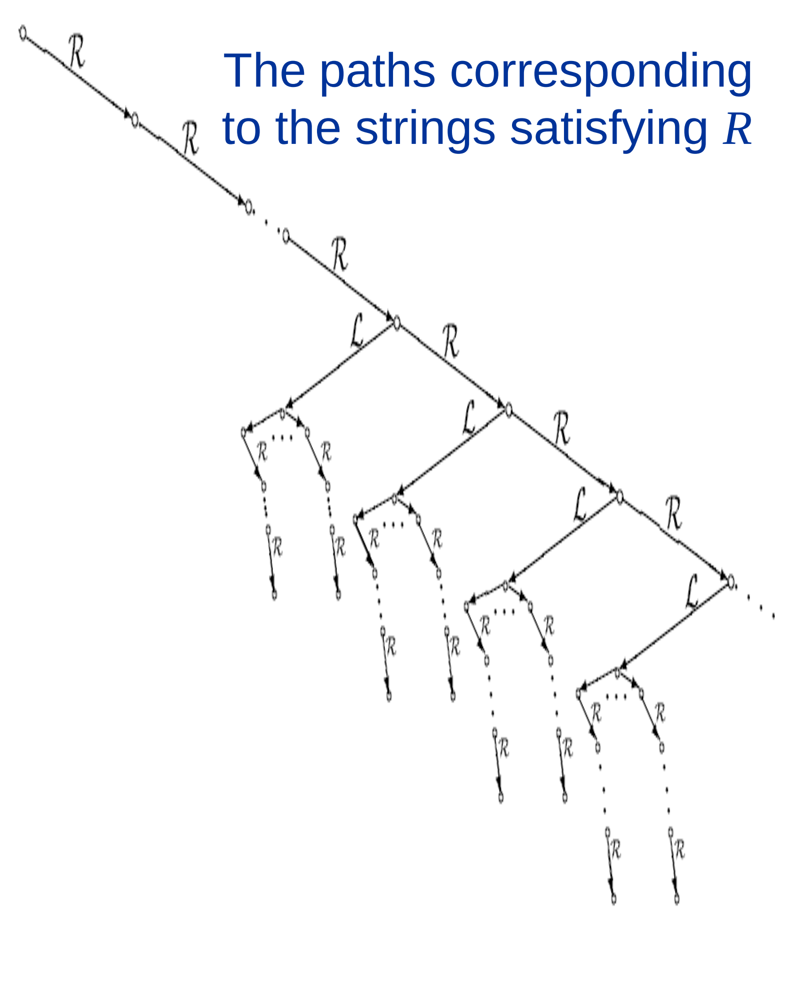
Similar trees





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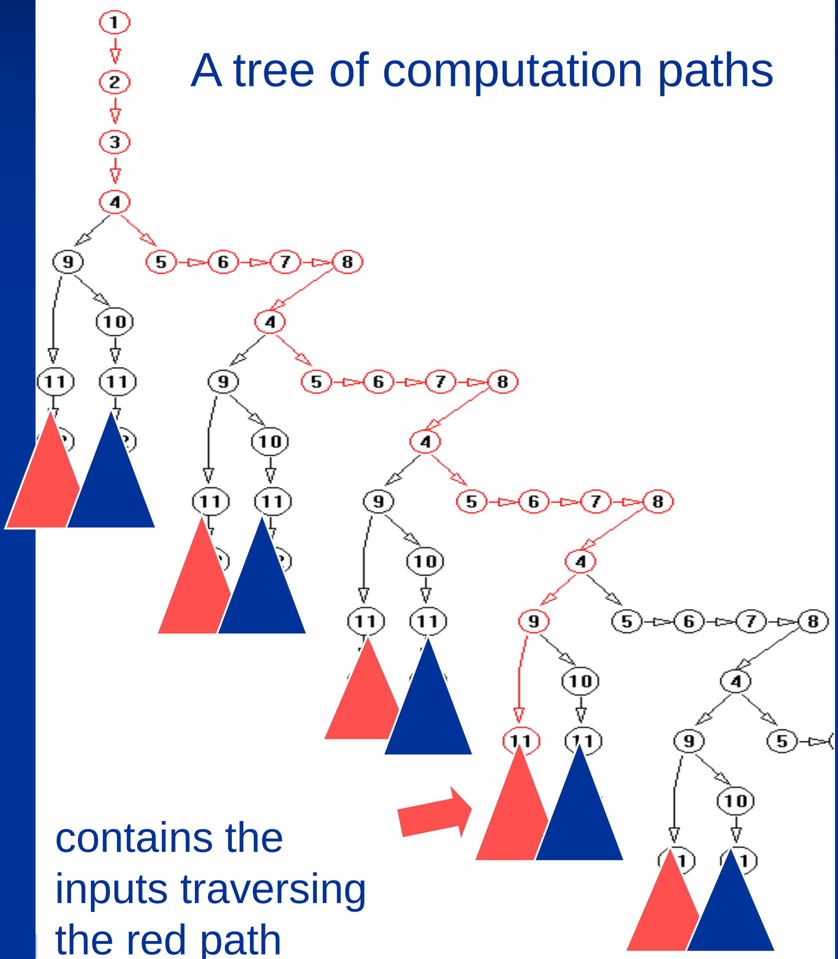
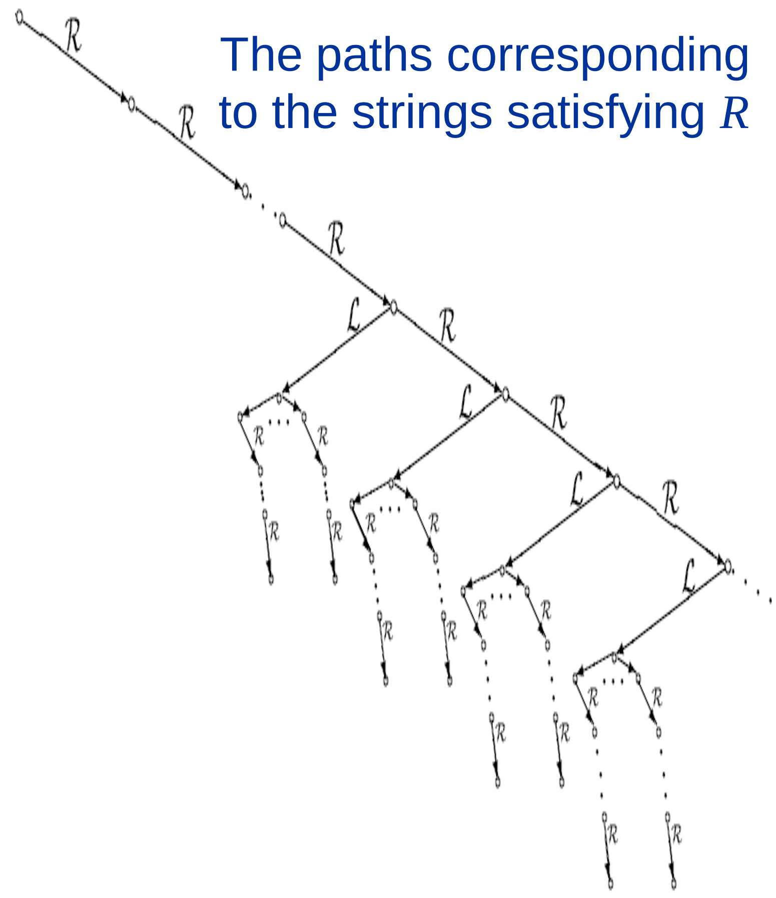
Similar trees

A tree of computation paths



contains the inputs traversing the red path

The paths corresponding to the strings satisfying R



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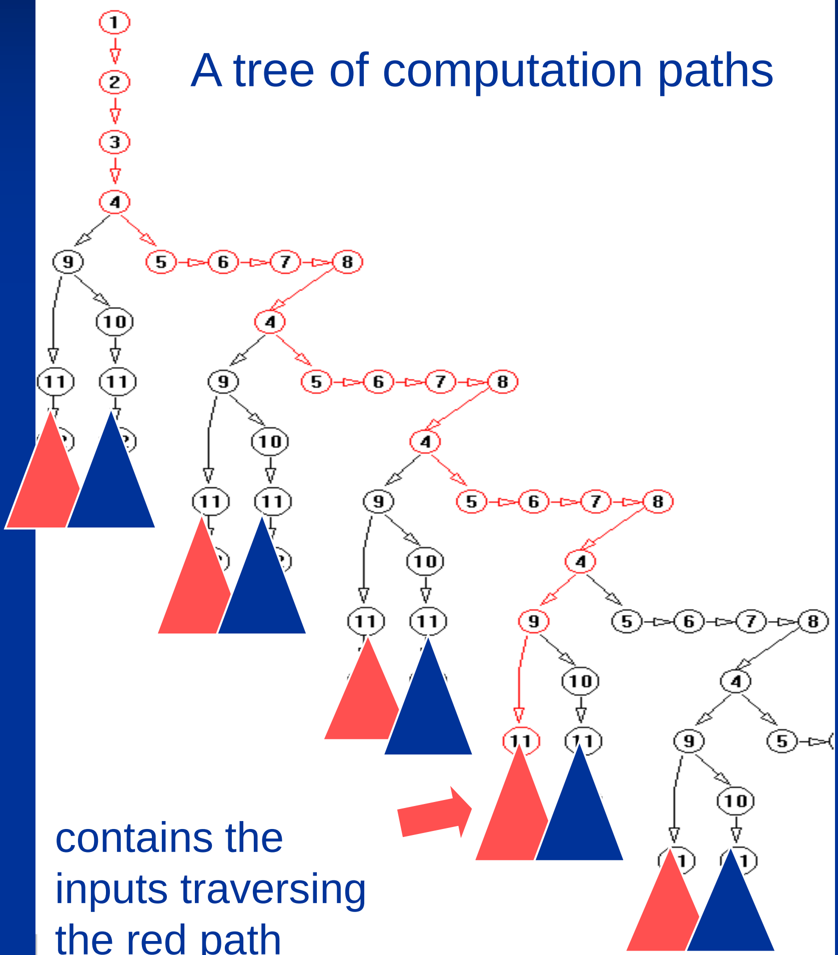

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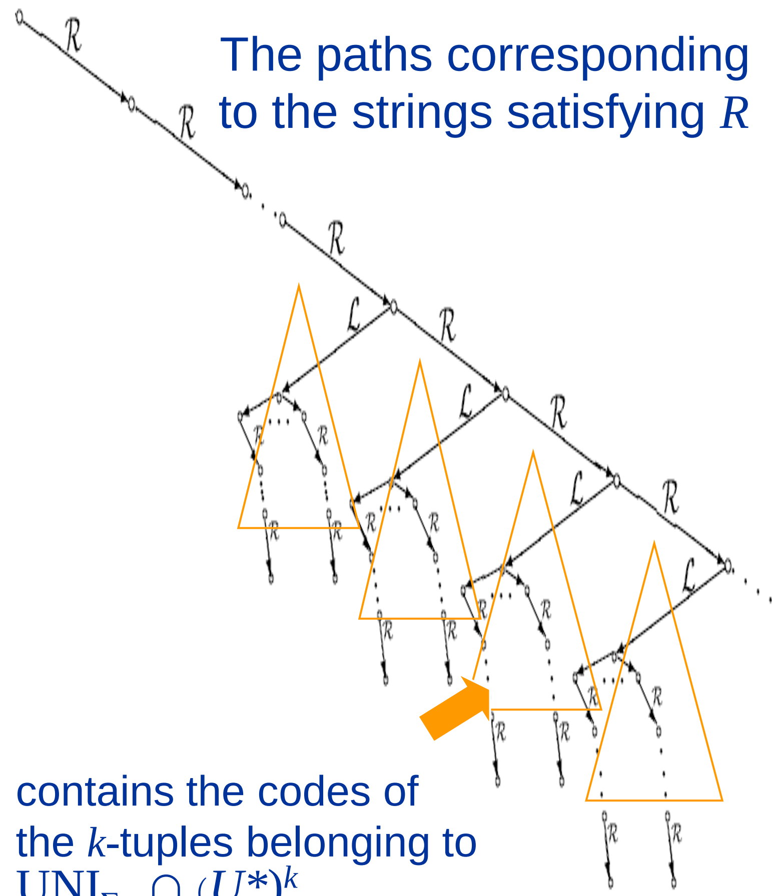
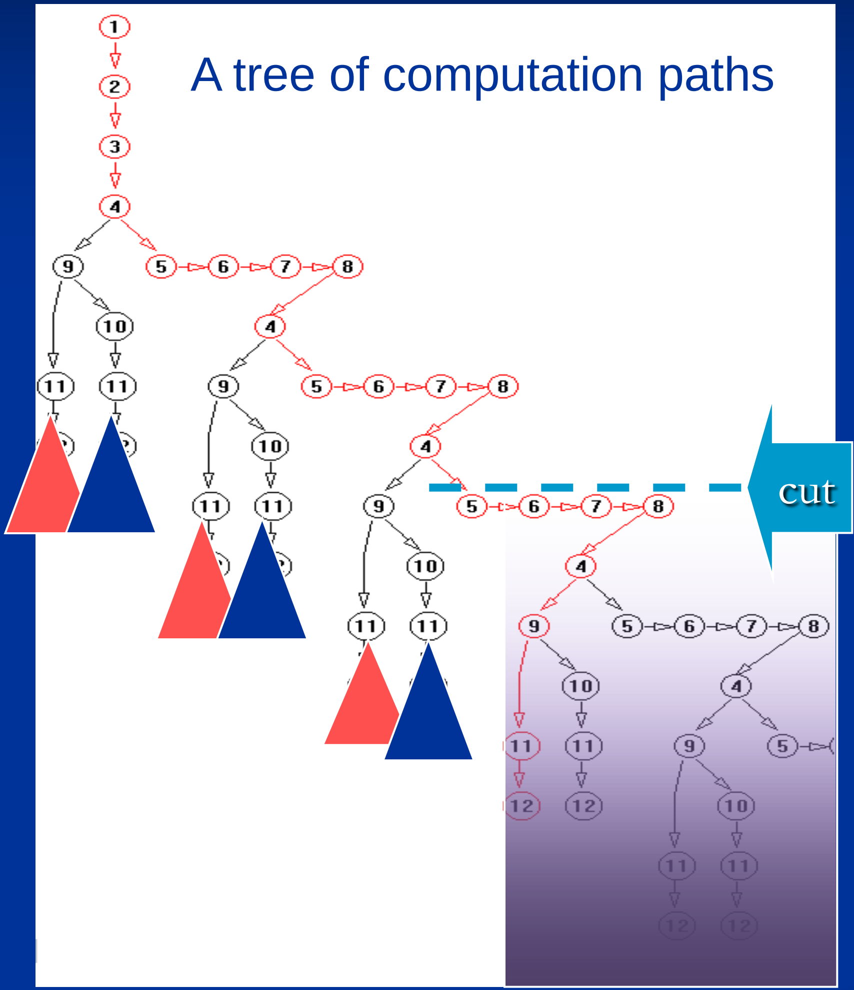
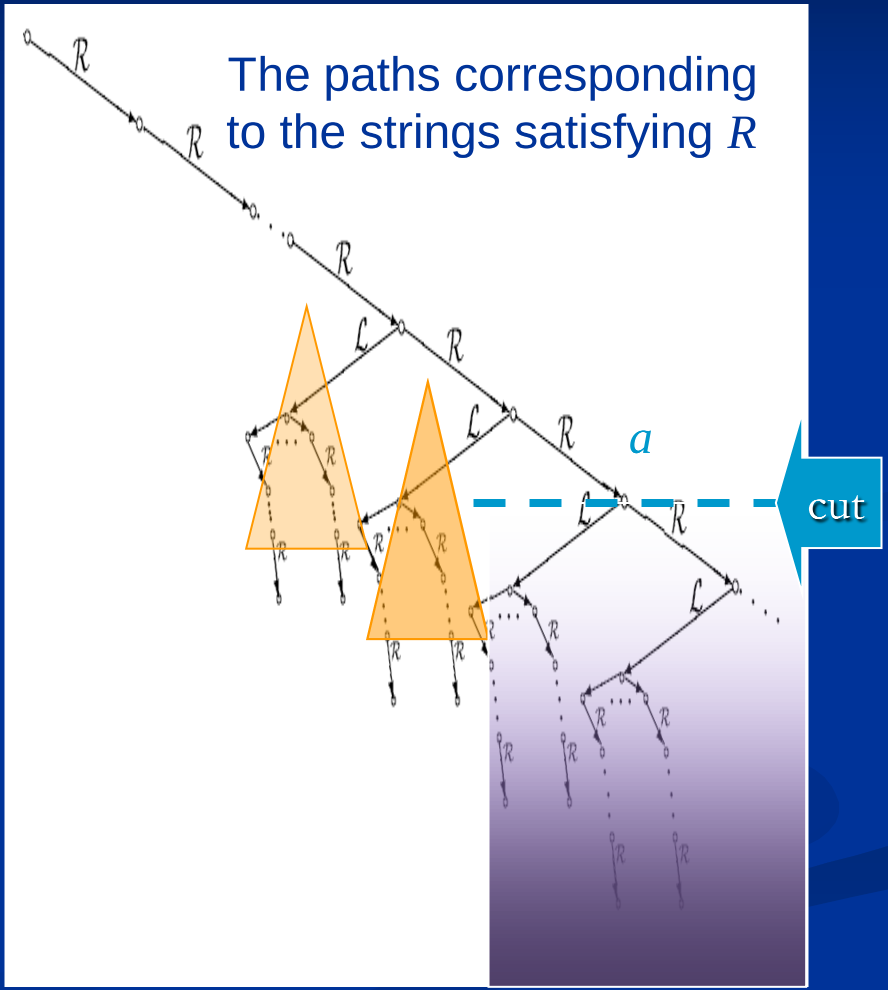


contains the codes of the k-tuples belonging to $\mathrm{UNI}_{\Sigma_R} \cap (U^*)^k$

Trees and polynomial time





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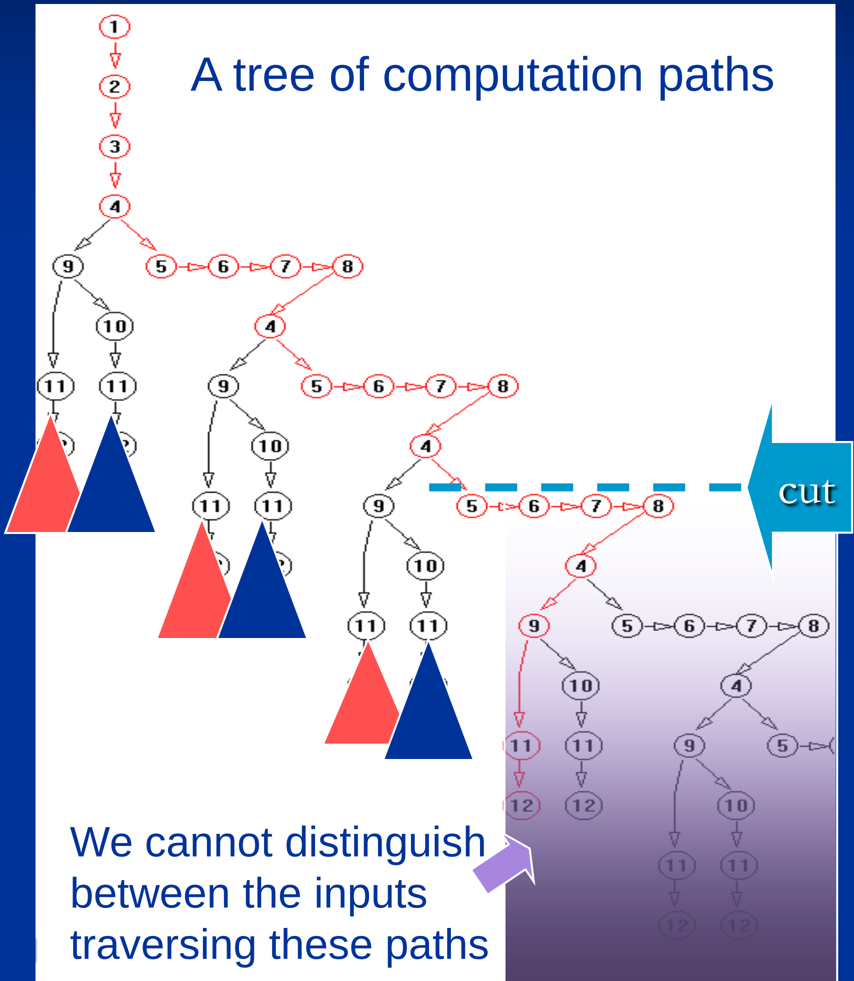

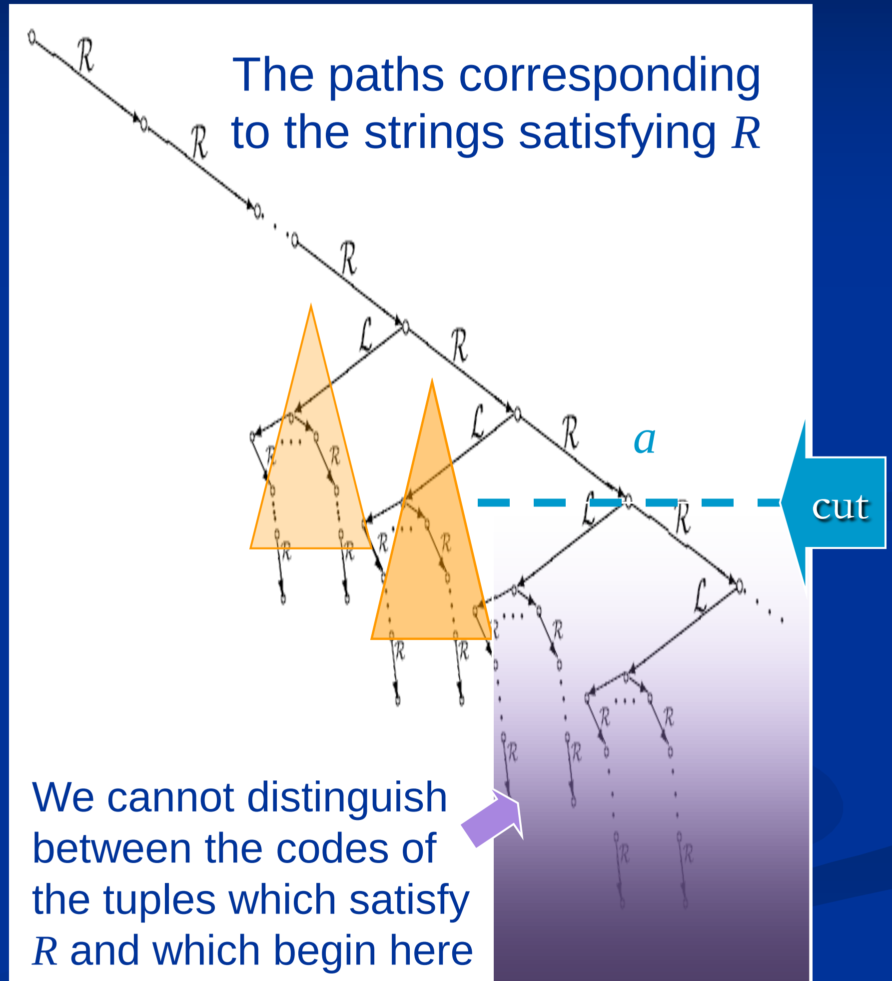

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$P_{\Sigma_R} = NP_{\Sigma_R}$

Proof of $P_{\Sigma_R} = NP_{\Sigma_R}$ by a reduction.

UNI $=\{(b,\ldots, b, x_1,\ldots, x_n, Code(M)) \mid \mathbf{x} \in (U^*)^\infty \ \& \ M$ is NP_{Σ_R}-mach. $\& \ M(\mathbf{x})\downarrow^t\}$

UNI = RES-UNI (the length of guesses can be restricted)

- Decompose $\{x_1,\ldots, x_n\}$ into equivalence classes,
- replace $x_1,\ldots, x_n$ by suitable short strings

such that possible chains are not destroyed.

SUB-UNI (short input strings) SUB-UNI ⊂ RES-UNI

- Transform the input tuple into a string,
- double the length,
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Output: a / b

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Why do we use strings?

Our goal:

- A relation R allows to decide whether $\boldsymbol{z} \in \mathcal{O}_\Sigma$.
- R can be defined recursively.

Problems:

- Each relation has a fixed arity.
- $\mathcal{O}_\Sigma$ contains tuples of any length.

- For many structures:
  The tuples of arbitrary length cannot be encoded by tuples of fixed length.

A solution:

- Strings.

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Appendix

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Structures over strings

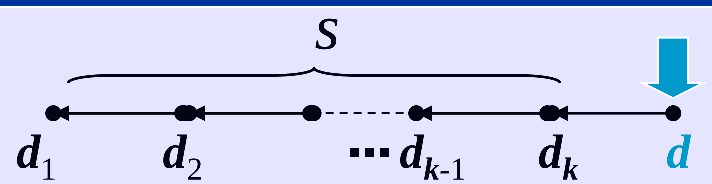
$$\Sigma = (U^* \,;\, \varepsilon, a, b, c_3, \ldots, c_u \,;\, add, sub_l, sub_r, f_1, \ldots, f_v \,;\, R_1, \ldots, R_w, R, =)$$

$s = d_1 \cdots d_k \in U^*$ stored in one register

$(d_1, \ldots, d_k) \in U^k \subset U^\infty$ stored in k registers

$add(s, d) = sd$ $\qquad sub_l(sd) = s$ $\qquad sub_r(sd) = d$

$s \in U^*, d \in U$



$s = d_1 \cdots d_{k-1} d_k$

$sd = d_1 \cdots d_k d$

$R_i \subseteq U^{n_i} \subseteq (U^*)^{n_i}$ and $R \subseteq U^*$

$f_i(s_1, \ldots, s_{m_i}) = \varepsilon$ if $|s_j| > 1$ for some j

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Structures over strings

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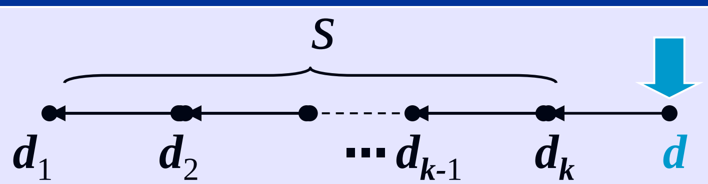

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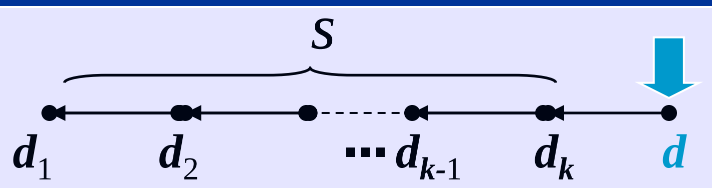

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Computation over strings

Example: $\Sigma = (\{a, b\}^* ; \varepsilon, a, b; add, sub_l; =)$

Definition: $s \subset_1 r \iff sub_l(r) = s$.

Lemma. For t steps of a machine holds:

1. The input values, the guesses, and the new computed values form maximal chains $s_1 \subset_1 \cdots \subset_1 s_k$.
2. The maximal chains form trees. Every tree has only one minimal element.
3. The predecessors $r \subset_1 s_1$ of the minimal elements s_1 are not computed.

Corollary:

The minimal elements can be replaced without changing the computation path.

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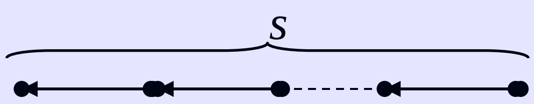

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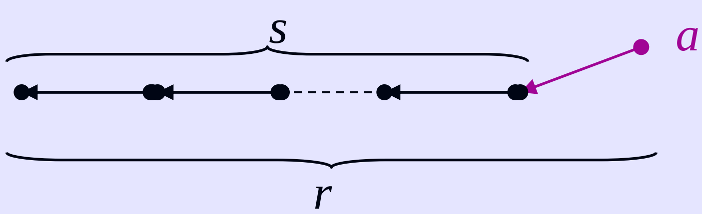

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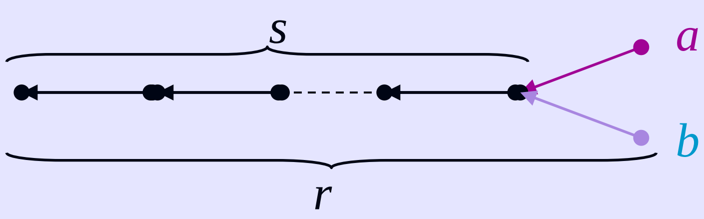

1) $r = sa$ $\quad add(s, a) = r$ $\quad sub_l(r) = s$

2) $r = sb$ $\quad add(s, b) = r$ $\quad sub_l(r) = s$

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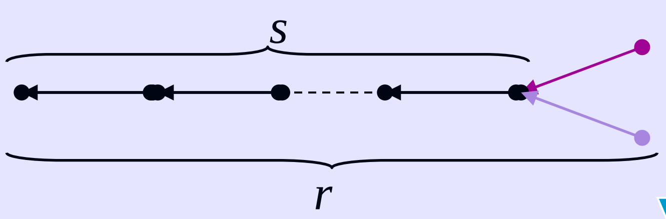
The minimal elements can be replaced without changing the computation path.

Computation over strings

Example: $\Sigma = (\{a, b\}^* ; \varepsilon, a, b; add, sub_l; =)$



1) $r = sa$ $add(s, a) = r$ $sub_l(r) = s$

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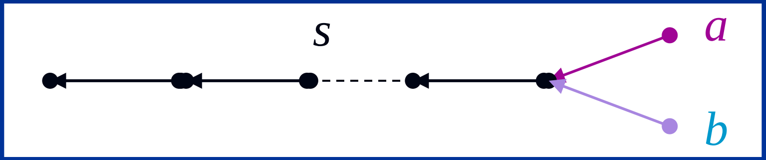

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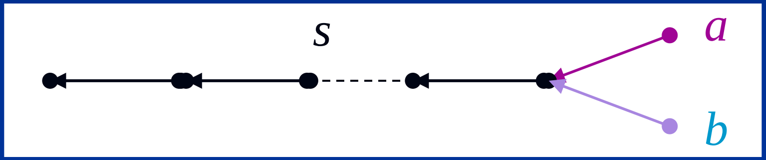

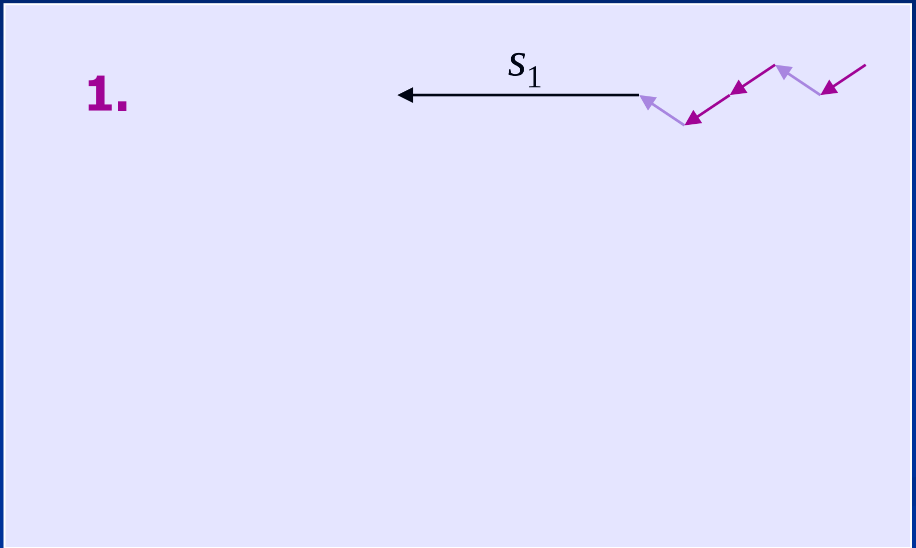

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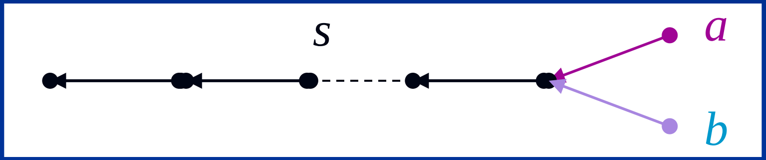

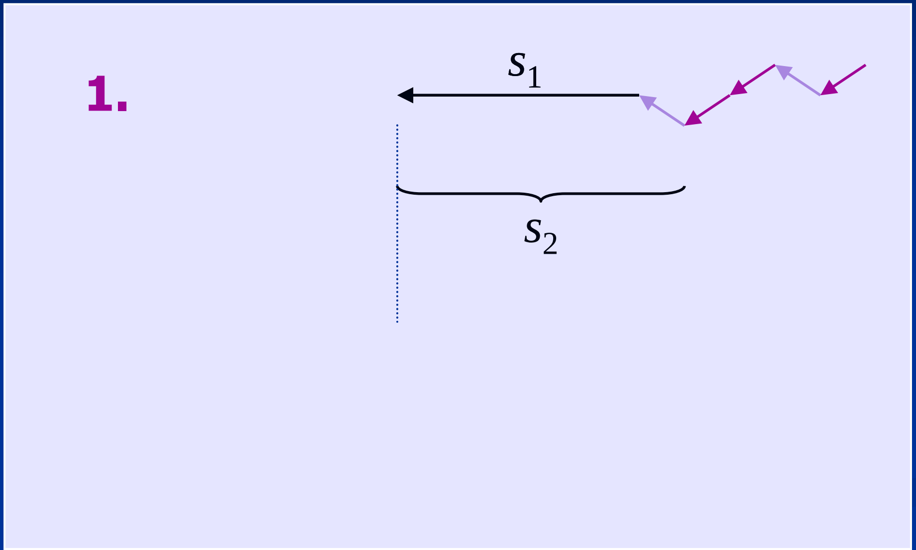

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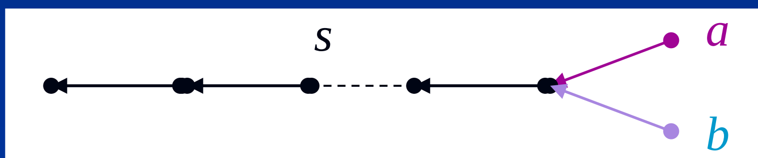

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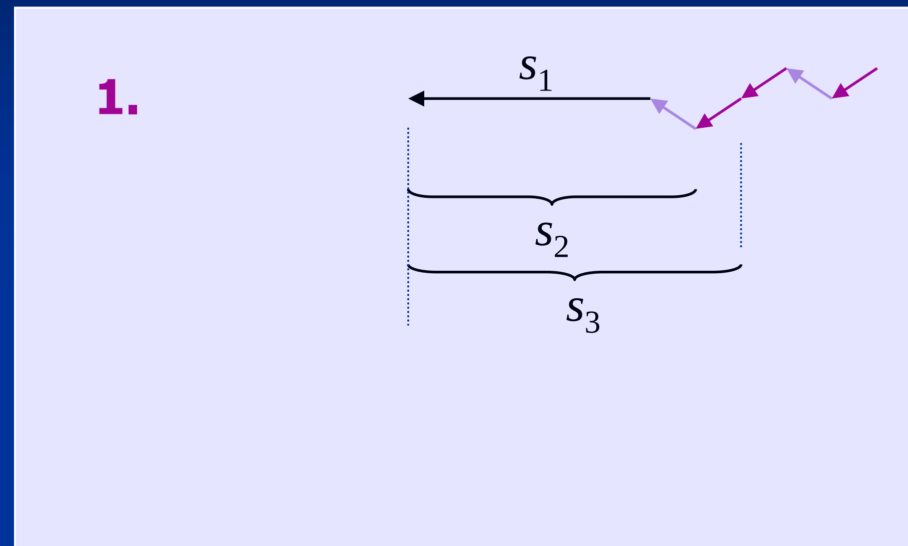

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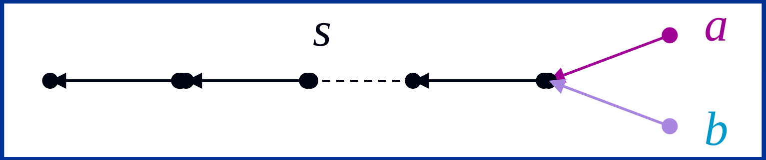

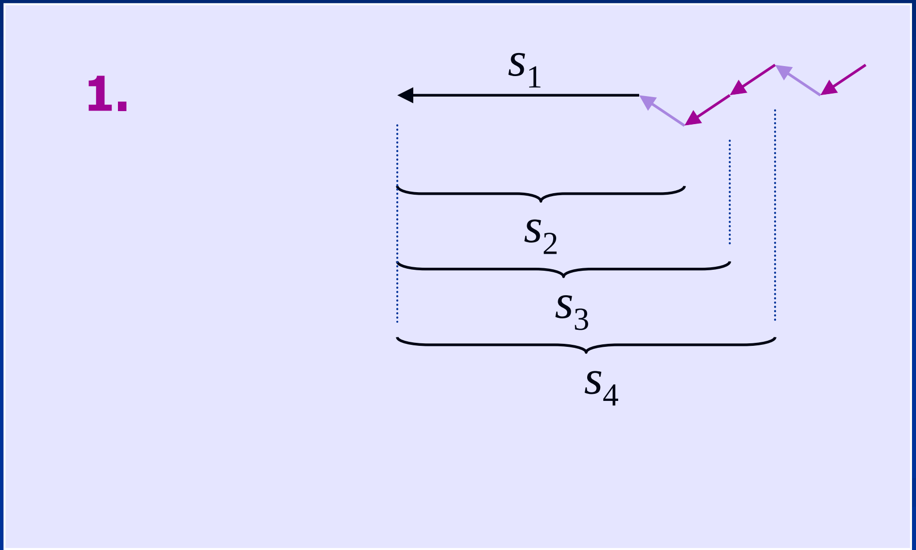

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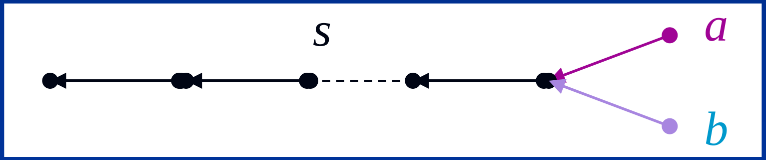

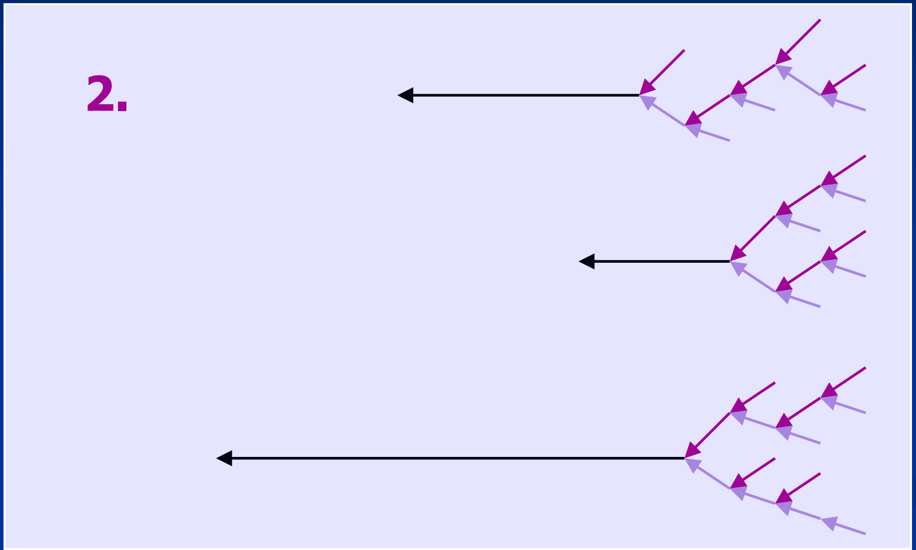

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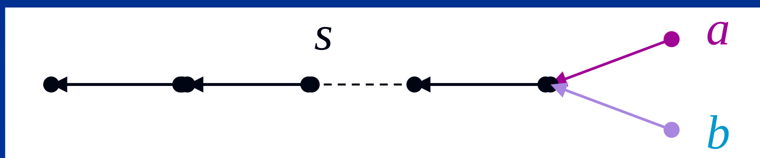

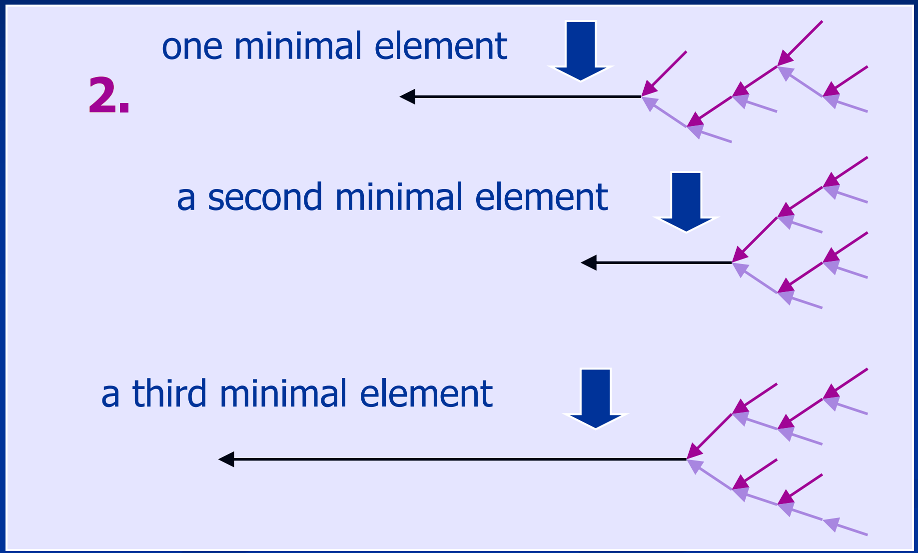

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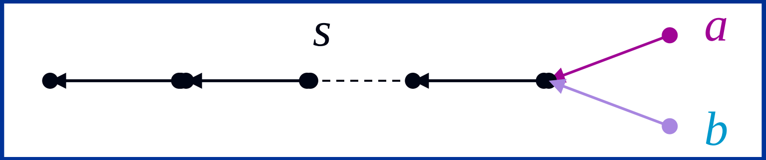

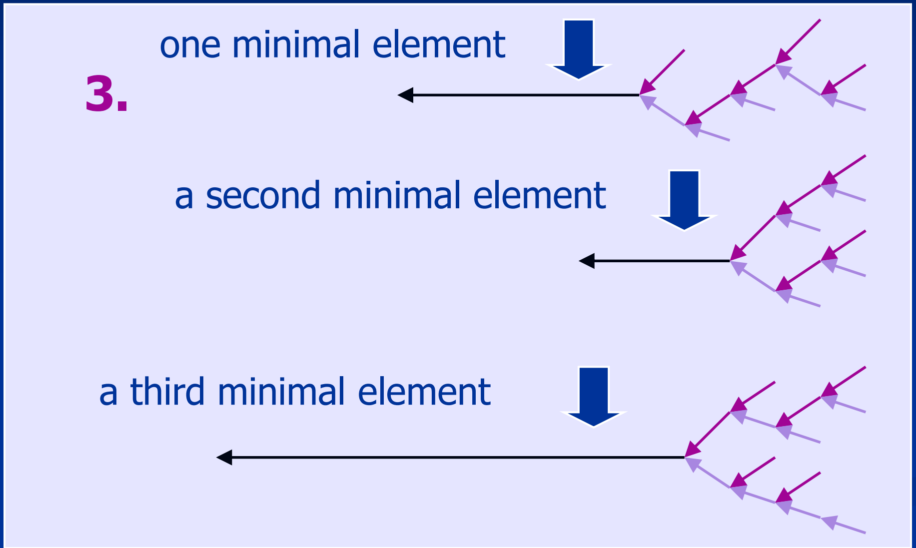

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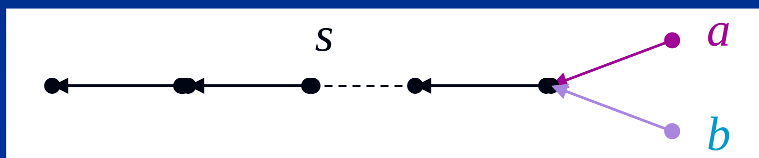

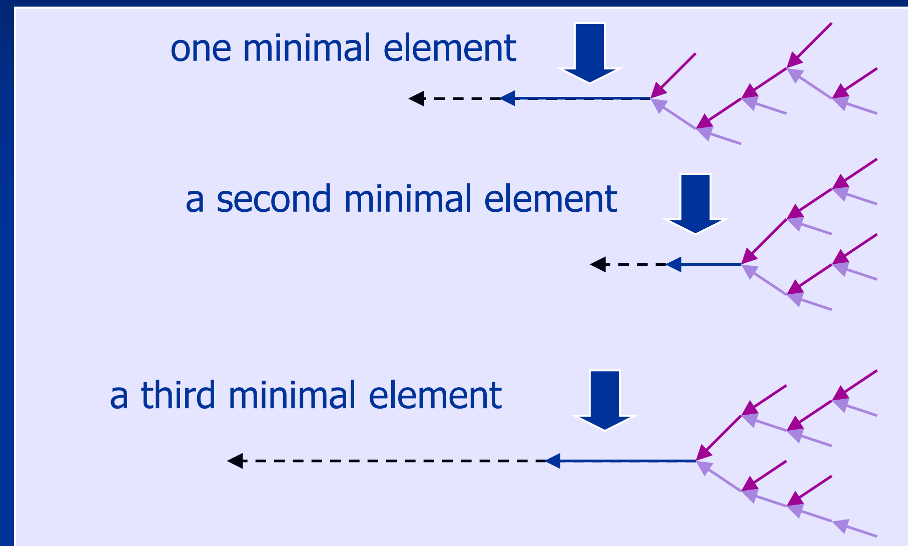

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Why do we pad the codes?

Our goal:

- Structures Σ with $\mathrm{NP}_\Sigma \subseteq \mathrm{DEC}_\Sigma$.

Problems:

- Arbitrary strings can be guessed.
- A new R could imply $\mathrm{H}_{\Sigma_R} \in \mathrm{NP}_{\Sigma_R} \setminus \mathrm{DEC}_{\Sigma_R}$ for the halting problem H_{Σ_R} .

Solution:

- Padding strings:

$$R(s) \Rightarrow (\exists r \in U^*)\,(\, s = ra^{|r|}\,).$$

It allows to replace

- long inputs and guesses
- by short strings over $\{a, b\}$.

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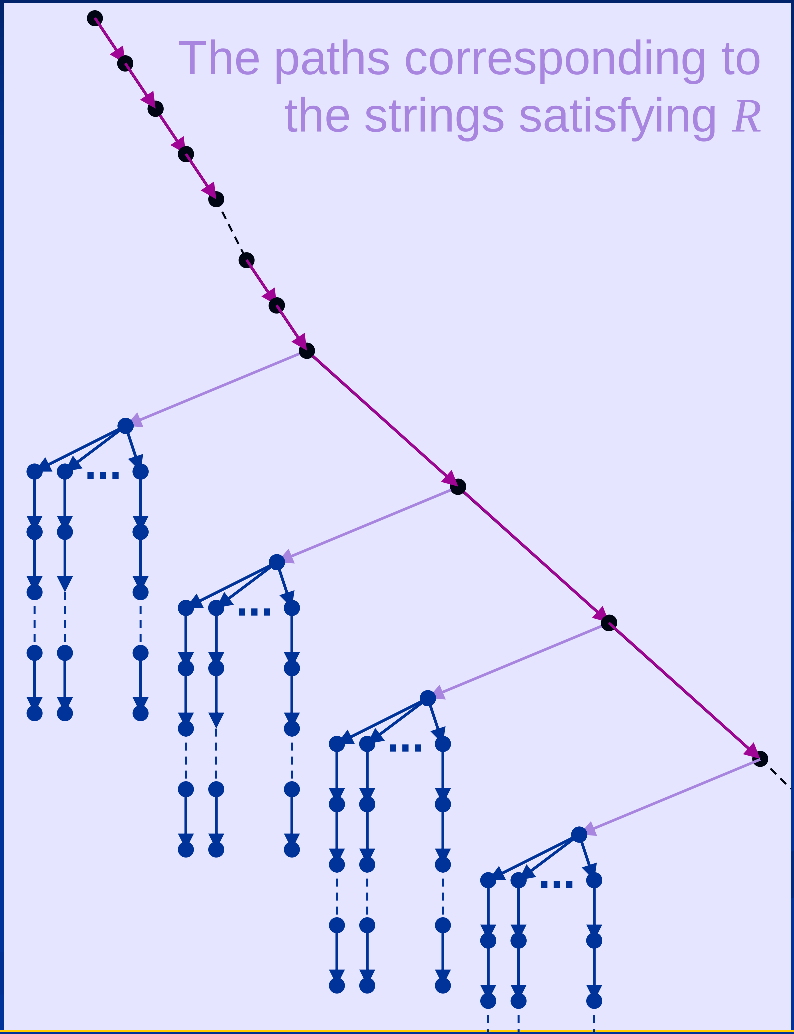

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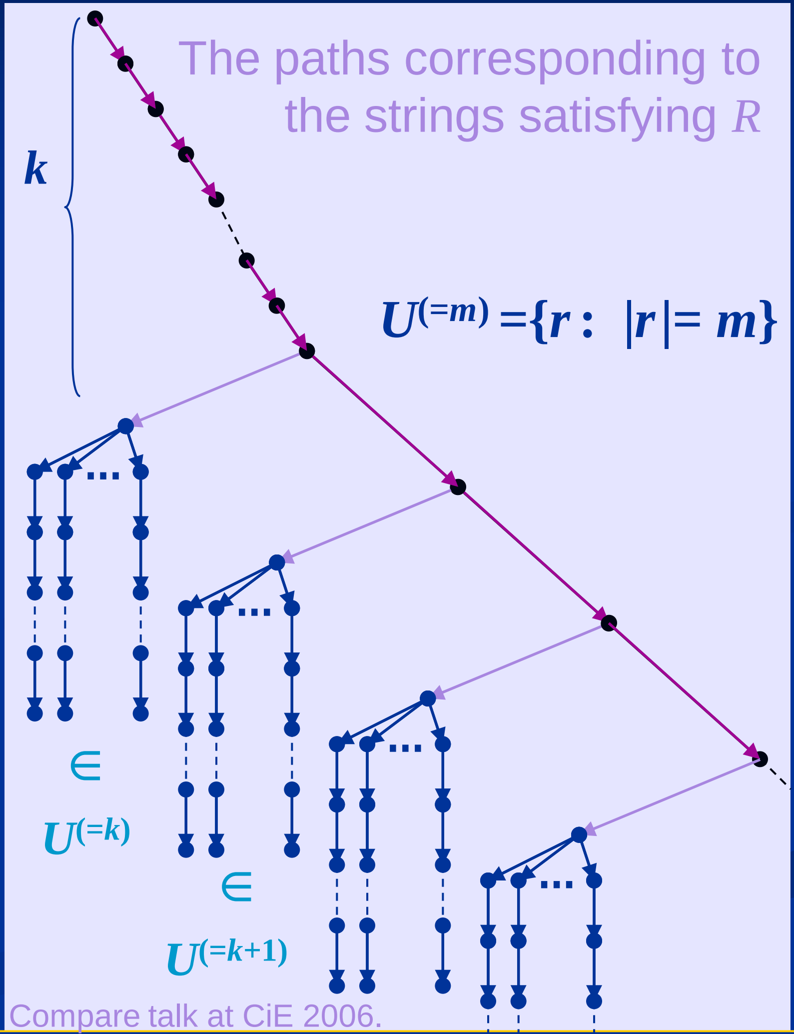

Compare talk at CiE 2006.

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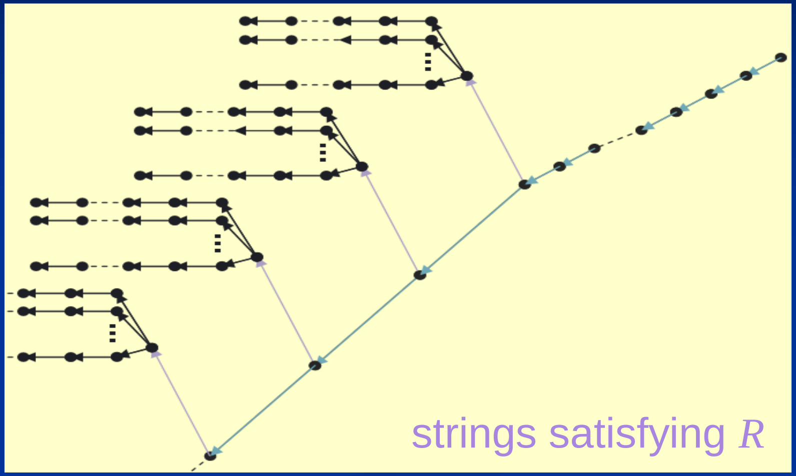

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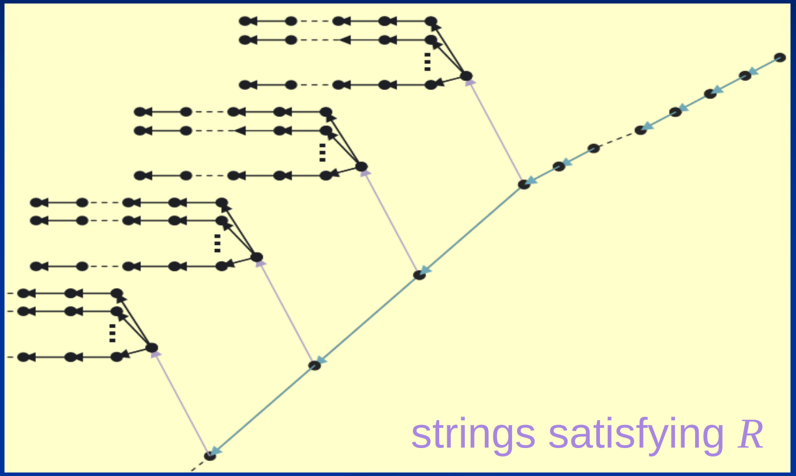

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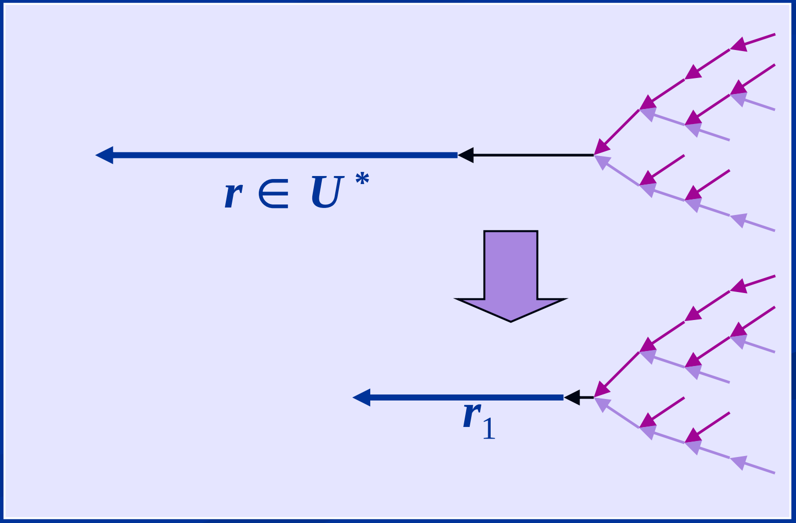

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The new relation R

$\Sigma = (U^* ; a, b, c_3, \ldots, c_u ; f_1, \ldots, f_v ; R_1, \ldots, R_w, =)$
Σ_R = Expansion of Σ by R

A universal oracle:

Let $\mathcal{W}_\Sigma \subset (U^*)^\infty$ with $P_\Sigma^{\mathcal{W}_\Sigma} = NP_\Sigma^{\mathcal{W}_\Sigma}$ (derived from $\mathcal{O}_\Sigma$).

The relation R:

$$r_1 \cdots r_k a^{|r_1 \cdots r_k|} \in R \iff (r_1, \ldots, r_k) \in \mathcal{W}_\Sigma$$

Theorem:

$P_{\Sigma_R} = NP_{\Sigma_R}$.

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$$\text{Let } W_\Sigma \subset (U^*)^\infty \text{ with } \mathrm{P}_\Sigma^{W_\Sigma} = \mathrm{NP}_\Sigma^{W_\Sigma}$$ (derived from O_Σ).

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(CiE 2007)

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Thank you very much!

Christine Gaßner

Vielen Dank auch

- für die Unterstützung bei der Vorbereitung von Präsentationen an

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