Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Winter 09/10
December 14, 2009
Exercise Sheet 9

Exercises for

Models of Set Theory II

33. Show that $\mathfrak{b} \leq cf(\mathfrak{d})$.

34. Assume that $F \subseteq \omega^\omega$ is an unbounded family which consists of increasing functions. Then for every function $g \in \omega^\omega$ and every infinite set $X \subseteq \omega$,

$$\exists f \in F \; \exists^\infty n \in X \; g(n) < f(n).$$

35. $\mathfrak{d}$ is equal to the smallest cardinal κ such that there exist families $F \subseteq \omega^\omega$ and $G \subseteq [\omega]^\omega$, both of size $\leq \kappa$ such that

$$\forall g \in \omega^\omega \; \exists f \in F \; \exists X \in G \; \forall^\infty n \in X \; g(n) < f(n).$$

For $X, Y \subseteq \omega$ write $X \subseteq^* Y$ iff $X - Y$ is finite and $Y - X$ is infinite.

A sequence $\langle X_\alpha \mid \alpha < \kappa \rangle$ in $[\omega]^\omega$ is called a tower if $X_\alpha \subseteq^* X_\beta$ for all $\alpha < \beta < \kappa$ and there is no $X \in [\omega]^\omega$ such that $X_\alpha \subseteq^* X$ for all $\alpha < \kappa$. Set

$$\mathfrak{t} = min\{card(T) \mid T \text{ is a tower}\}.$$

36. Show that $\omega < \mathfrak{t} \leq \mathfrak{b}$.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at December 21, 2009.