Mathematisches Institut
der Universität Bonn
P. Koepke, B. Irrgang

Winter 09/10
December 7, 2009
Exercise Sheet 8

Exercises for

Models of Set Theory II

The sets

$$[s] = \{f \in \omega^\omega \mid s \subseteq f\}$$

with $s \in \omega^{<\omega}$ form a basis for a topology on ω^ω. The set ω^ω endowed with this topology is called the Baire space.

The sets

$$[s] = \{f \in 2^\omega \mid s \subseteq f\}$$

with $s \in 2^{<\omega}$ form a basis for a topology on 2^ω. The set 2^ω endowed with this topology is called the Cantor space.

29. Define an injective map j from the Baire space into the Cantor space in such a way that the Baire space is under j homeomorphic to the subspace of the Cantor space which consists of all not eventually constant functions.

30. For $f \in 2^\omega$ let $k(f) = \sum_{n\in\omega} f(n)\ 2^{-(n+1)}$. Show that k is a continuous surjection of the Cantor space on the unit interval $[0, 1]$.

31. Let X be a subset of the Baire space. Show that $j[X]$ is meager iff X is.

32. Let X be a subset of the Cantor space. Show that $k[X]$ is meager iff X is.

Every problem will be graded with 8 points.

Please hand in your solutions during the lecture at December 14, 2009.